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CONCRETE CONTAINMENT STRUCTURE SHELL STRIP EXPERIMENTS AND SINGLE ELEMENT MODELLING OF SHELL ELEMENTS SUBJECTED TO SHEAR

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ABSTRACT

This paper discusses a novel three-dimensional single element method, called the 3D Single Element Method (3D SEM), capable of predicting the combined in-plane and out-of-plane shear response of shells. The paper outlines the theoretical framework of the model and compares the results to shell tests in the literature. The method uses the truss analogy for shear applied in three-dimensions to convert the eight sectional resultants into equivalent stresses that act on a representative triaxial element. The equations of the Modified Compression Field Theory in three-dimensions are then applied to determine the full load-deformation response of the triaxial element, capable of transmitting three shear stresses and three axial stresses. To further verify and validate the model, the paper then presents the results from three recently conducted experiments on reinforced concrete shell strips, representative of containment walls. The large-scale experiments, measuring 4877 x 610 x 610 mm were instrumented with three-dimensional digital image correlation (DIC) equipment and were tested to failure. The experiments investigated the influence of axial compression and axial tension on the shear behavior of the members. The experiments were compared to predictions from the 3D SEM. The results indicate that the 3D SEM is capable of capturing the complex interaction of combined in-plane and out-of-plane shear on reinforced concrete shell elements. The results from the experimental series demonstrate the influence of axial stresses on the behavior of shear critical shell strips and that the 3D SEM is capable of predicting the response of these members.

INTRODUCTION

In the design and assessment of nuclear facilities, reinforced concrete shell structures are often subjected to complex loading combinations that need to be considered. In particular, concrete shell structures such as nuclear containment walls can be subjected to combined loading that may occur as a result of seismic activity or an overpressure event. In these situations, the structures are simultaneously subjected to various combinations of the eight stress resultants, including combinations of in-plane and out-of-plane shear (see Figure 1).

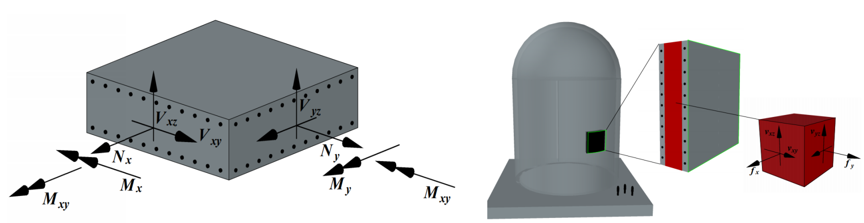


Figure 1. Shell subjected to eight stress resultants (left), nuclear containment structure and selected element for analysis (right).

Previous research has shown that the interaction between combined in-plane and out-of-plane shear is complex (Adebar, 1989; Adebar and Collins, 1994 and Proestos, 2018). While finite element tools can provide excellent results, they can require substantial time to be used effectively. Thus, in the design and assessment of shell structures, simple analysis methods are needed for predicting the shear response of members subjected to combined loads. Establishing simplified assessment approaches for shells can assist in the verification and validation of more complex tools or to train algorithms that may be used as a part of structural inspections and other nuclear facility operations.

This paper discusses a novel approach, called the 3D Single Element Method (3D SEM), capable of predicting the combined in-plane and out-of-plane shear response of shells. The paper outlines the theoretical framework of the model and compares the results to shell tests in the literature. The method uses the truss analogy for shear applied in three-dimensions to convert the eight sectional resultants into equivalent stresses that act on a representative triaxial element. The equations of the Modified Compression Field Theory (MCFT) in three-dimensions are then applied to determine the full load-deformation response of the triaxial element, capable of transmitting three shear stresses and three axial stresses (Vecchio and Collins, 1986 and Adebar, 1989).

To further verify and validate the model, the paper then presents the results from three recently conducted experiments on reinforced concrete shell strips representative of containment walls. The large-scale experiments, measuring 4877 x 610 x 610 mm were instrumented with three-dimensional digital image correlation (DIC) equipment and were tested to failure. The experiments investigated the influence of axial compression and axial tension on the shear behavior of the members. The experiments were compared to predictions from the 3D SEM. The results indicate that the 3D SEM is capable of capturing the complex interaction of combined in-plane and out-of-plane shear on reinforced concrete shell elements. The results from the experimental series demonstrate the influence of axial stresses on the behavior of shear critical shell strips and that the 3D SEM is capable of predicting member response.

THE 3D SINGLE ELEMENT METHOD FOR PREDICTING SHEAR RESPONSE OF SHELLS

The 3D SEM is an extension of the 2D Single Element Method developed by Collins (Collins and Mitchell, 2014) and Proestos (2018). The 2D Single Element Method is an approach to predict the shear response of reinforced concrete beams in two-dimensions by representing beam members with a single two-dimensional membrane in the web of the member (see Figure 2). This is consistent with the AASHTO LRFD Code, the Canadian CSA A23.3 Code and the *fib* Model Code, however rather than using simplified relationships to predict shear response, the full constitutive relationships of the MCFT can be applied (AASHTO LRFD, 2020; CSA A23.3-19, 2019 and *fib* Model Code, 2010). More information on the 2D Single Element Method can be found elsewhere (Proestos, 2018).

The objective of the 3D SEM is to create a simplified method for estimating the response of shear critical reinforced concrete shell elements subjected to combinations of the eight stress resultants using a single triaxial MCFT element while ensuring the influence of all eight stress resultants are considered. The concept of the 3D SEM is that the response of a shear critical shell element can be estimated from the eight sectional stress resultants (applied loads), section geometry, reinforcement properties and by applying the three-dimensional constitutive relationships of the MCFT at a representative point in the structure. Specifically, the equivalent stresses, section geometry and reinforcement quantities are determined for the section. The representative three-dimensional element is located at a single point at the mid-depth of the section and therefore has no physical dimensions or moments acting on it. Thus, the fundamental challenge in applying the method is determining how to translate the stress resultants and properties of a shell element to the single triaxial MCFT element. See Figure 2.

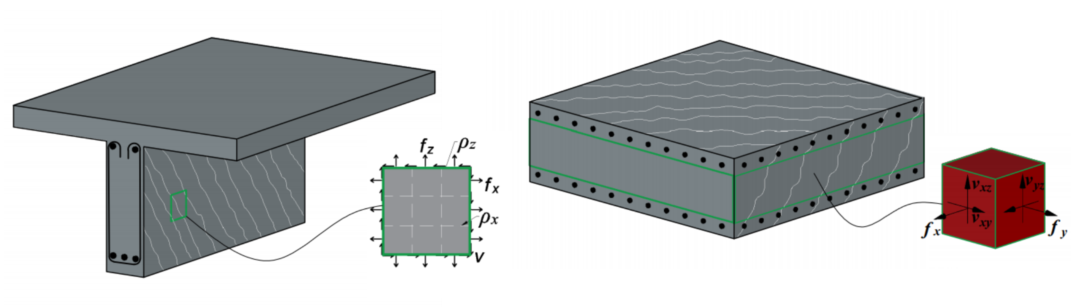


Figure 2. Representation of 2D Single Element Method (left), representation of 3D SEM (right).

Additionally, the 3D SEM uses the complete constitutive relationships of the MCFT for cracked reinforced concrete in three-dimensions (Vecchio and Collins, 1986). These equilibrium, compatibility and stress-strain relationships relate the three axial stresses and three shear stresses (f_x, f_y, f_z, v_{xy}, v_{yz} and v_{xz}) to the three axial strains and three shear strains (ε_x, ε_y, ε_z, γ_{xy}, γ_{yz} and γ_{xz}) at a point in space. The fundamental assumption of the MCFT is that the directions of principal concrete compressive strains are coincident with the directions of principal concrete compressive stresses. This assumption, used in conjunction with stress-strain relationships, in compression and tension, provide a basis for determining the stiffness matrix that relates the strains and stresses at a point. A summary of the MCFT in three-dimensions is provided in Figure 3 and more detail explanation can be found elsewhere (Adebar, 1989; Bentz, 2000 and Proestos 2018).

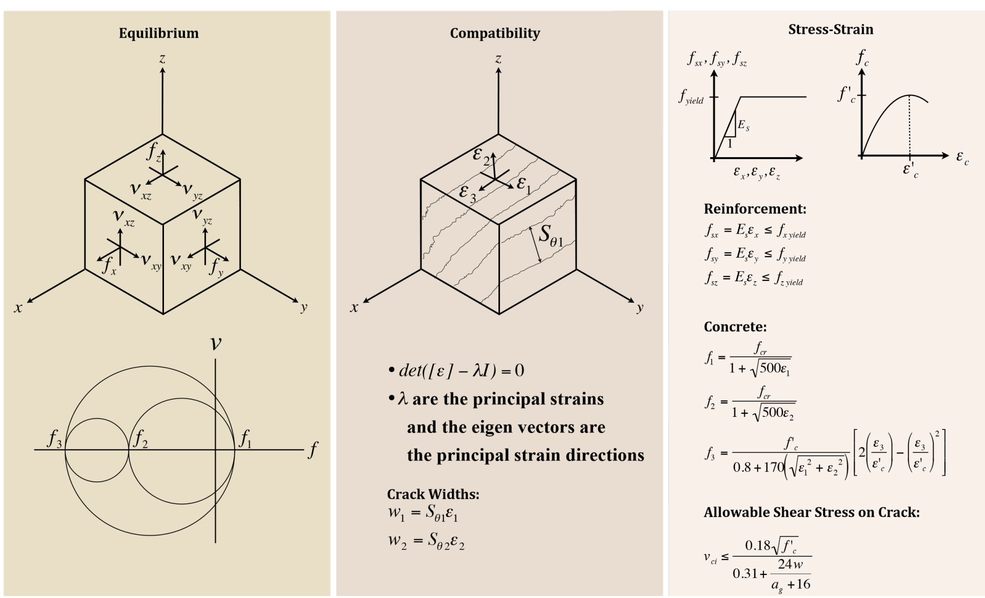
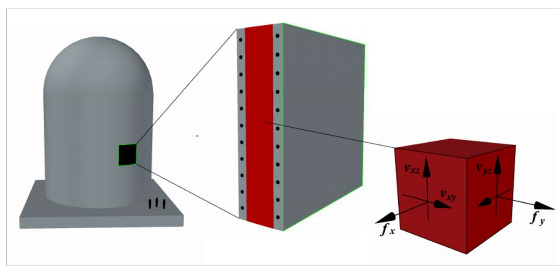


Figure 3. Summary of three-dimensional relationships of the MCFT.

While a parabola is shown in Figure 3 for convenience, the more complete modified Popovics constitutive relationship is used for the concrete constitutive relationship in the 3D SEM (Collins and Mitchell, 1991). The compression softening relationship proposed by Bentz et al. (2006) is used in the analysis. This relationship accounts for the phenomena that the strength of concrete in compression will be less than the uniaxial cylinder strength as a result of coexisting tensile strains in one or two of the other principal directions. Thus, when one or more of the principal stresses are tensile, the compression softening relationship is used. When the principal strains are tensile, the tension stiffening relationship is used to determine the tension carried on average by the cracked concrete. If the principal strains do not exceed the concrete cracking strain in a particular direction, a linear elastic response is used. For the reinforcing steel, an elastic-plastic response is used. A three-dimensional crack check, originally developed by Adebar (1989) and Bentz (2000), is used to verify that the shear stresses can be transmitted across the cracks and ensures that the shear stresses do not exceed the maximum shear stress on each crack, defined as $v_{ci,\,max}$.

The process to assesses a shell element using the 3D SEM is presented in Figure 4. The first step in the process is to determine the section to conduct the analysis and sectional stress resultants. As is recommended in design codes for members in two-dimensions, the sectional stress resultants can be taken at a section a distance d_v away from the face of the shell support, or load (AASHTO LRFD, 2020; CSA A23.3-19, 2019 and *fib* Model Code, 2010). The length d_v is defined as $0.9d$, where d is the depth of the flexural tension reinforcement and h is the specimen height. Second, the equivalent stresses are determined from the sectional stress resultants for a unit shell element using the equations in Figure 4 for f_x, f_y, v_{xy}, v_{yz}, and v_{xz}. The first term in the equations accounts for the applied axial or shear stresses. The second term in the equations is an additional term that accounts for the straining of the element due to the applied moments acting on the section. Since the elements are considered slender the clamping stress, f_z, is taken as zero. Step three involves determining the reinforcement ratios for the unit shell. The 3D SEM is capable of predicting the response of heavily or lightly reinforced members, thus if there is no reinforcement in a particular direction, the reinforcement quantity should be taken as zero in that direction. It should also be noted that the reinforcement ratios are taken over a height d_v.

Three-Dimensional Single Element Method Procedure



STEP 1. Determine desired section and applied loads.

STEP 2. Determine equivalent stresses. ***M/V*** taken at d_v away from the face of the load, shell, or support.

$$f_x = \frac{N_x}{h} + \frac{M_x}{V_{xz} d_v} v_{xz}$$

$$f_y = \frac{N_y}{h} + \frac{M_y}{V_{yz} d_v} v_{yz}$$

$$v_{xy} = \frac{V_{xy}}{h} + \frac{M_{xy}}{V_{xy} d_v}\left(\frac{V_{xy}}{h}\right)$$

$$v_{yz} = \frac{V_{yz}}{d_v}$$

$$v_{xz} = \frac{V_{xz}}{d_v}$$

STEP 3. Determine reinforcement ratios ρ_x, ρ_y, ρ_z.

$$\rho_x = \frac{A_{s,x}}{1000 * d_v} \qquad \rho_y = \frac{A_{s,y}}{1000 * d_v}$$

$$\rho_z = \frac{A_v}{1000 * 1000}$$

or taken as zero if no reinforcement in the respective direction

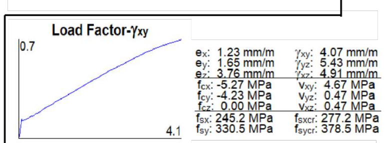
STEP 4. Determine the crack spacing s_x, s_y, s_z.

$$s_x = \textit{reinforcement spacing}$$

$$s_y = \textit{reinforcement spacing}$$

$$s_z = \begin{cases} d_v \; if \; \frac{A_v f_{y,z}}{b_w s} \geq 0.08\sqrt{f'_c} \\ 5h \; if \; \frac{A_v f_{y,z}}{b_w s} < 0.08\sqrt{f'_c} \end{cases}$$

STEP 5. Conduct MCFT analysis of single element using 3D MCFT procedure.



$v_{xy} = 4.01$ MPa

$v_{xz} = v_{yz} = 0.47$ MPa

$$\frac{v_{xy,exp}}{v_{xy,Triax}} = \frac{v_{xz,exp}}{v_{xz,Triax}} = 0.89$$

Example in Triax (PS4):

$h = 285$ mm	$f_{yx} = f_{yy} = 593$ MPa
$d = 222$ mm	$A_{s,x} = 18{,}000$ mm^2
$d_v = 200$ mm	$A_{s,y} = 12{,}000$ mm^2
$f'_c = 30.6$ MPa	$b_{width} = 2155$ mm

$$f_x = \frac{N_x}{h} + \frac{M_x}{V_{xz} d_v} v_{xz} = 6.72 \text{ MPa}$$

$$f_y = \frac{N_y}{h} + \frac{M_y}{V_{yz} d_v} v_{yz} = 6.72 \text{ MPa}$$

$$v_{xy} = \frac{V_{xy}}{h} + \frac{M_{xy}}{V_{xy} d_v}\left(\frac{V_{xy}}{h}\right) = 6.34 \text{ MPa}$$

$$v_{yz} = \frac{V_{yz}}{d_v} = 0.643 \text{ MPa}$$

$$v_{xz} = \frac{V_{xz}}{d_v} = 0.643 \text{ MPa}$$

$\rho_x = \frac{18{,}000}{2155 * 200} = 4.18\%$

$\rho_y = \frac{12{,}000}{2155 * 200} = 2.78\%$

$\rho_z = 0.0\%$

$s_x = 72$ mm (x-reinforcement spacing)

$s_y = 108$ mm (y-reinforcement spacing)

$s_z = 5h = 1425$ mm

Figure 4. Summary of 3D SEM procedure.

In step four, the reinforcement spacing is used as the crack spacing in the x- and y-directions. In the z-direction, if there is at least minimum transverse reinforcement, the crack spacing is taken as d_v. If there is less than minimum or no transverse reinforcement the crack spacing is taken as a large number, such as $5h$. Step five involves using these input parameters to determine the entire stress-strain response using the full MCFT relationships. For this paper an in-house implementation of the three-dimensional MCFT was used, alternatively other available tools, such as program Triax-2000 can be used (Bentz, 2000). Figure 4, also provides an example of the inputs and outputs of the procedure for specimen PS4, an experiment conducted by Proestos (2018), described elsewhere. More information on the details of the 3D SEM process can be found elsewhere (Stuart, 2021).

Figure 5 shows the results of the 3D SEM used to predict the response of the SP series of shell element tests, tested at the University of Toronto by Adebar and Collins (1994). The experimental series investigated the response of shell elements subjected to combined in-plane and out-of-plane shear. Figure 5 also shows, the predictions from Shell II a simplified three-layer nonlinear finite element tool developed by Proestos (2018) for reinforced concrete shell elements. As discussed previously, by default, the 3D SEM determines the reinforcement ratios for the shell elements taken over a depth d_v, however in certain circumstances, such as in situations governed by in-plane actions, taking the reinforcement ratios over d_v increases the strength of the elements. Therefore, for comparison, Figure 5 also shows the predictions when the reinforcement ratios are calculated over a depth of h. As can be seen the results are fairly consistent when governed by yielding of the transverse reinforcement (such as near specimen SP9, SP7 and SP3 but gives somewhat different predictions for members governed by in-plane response such as SP8. As can be seen, the results of the 3D SEM reasonably predict the experiments and agree well with Shell II. In particular the asymmetrical interaction diagram, that occurs as a result of the compression field associated with the out-of-plane shear either aligning or acting perpendicular to the compression field associated with the in-plane shear, is well predicted. The ability for the 3D SEM to predict this interaction is impressive given that the analysis only uses a single element and takes a few seconds to run. The results near SP4 are somewhat over-predicted, however, these predictions are governed by first cracking and can be somewhat sensitive to the assumptions made for the how flexural stresses influence the results.

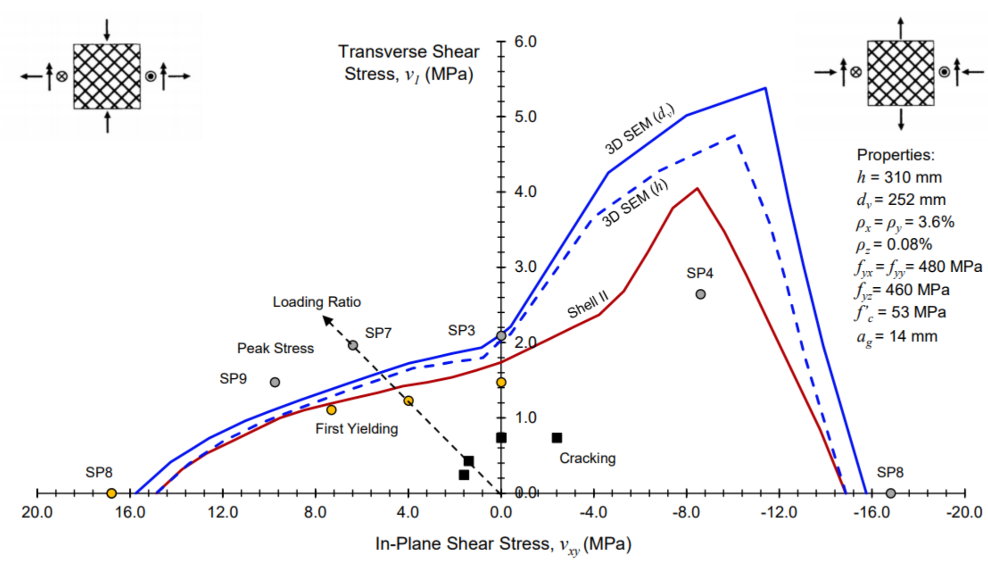


Figure 5. In-plane versus out-of-plane shear interaction diagram for the SP series of tests and 3D SEM predictions.

CONCRETE CONTAINMENT STRUCTURE SHELL STRIP EXPERIMENTS

To further verify and validated the 3D SEM and to investigate the response of members subjected to out-of-plane shear combined with axial stresses, experimental testing was conducted on six reinforced concrete elements. While six experiments were conducted by Stuart (2021), three of the specimens which contained shear reinforcement (SSA1, SSA3 and SSA5) will be examined in this paper. These three tests were 610 x 610 x 4877 mm. As is typical of nuclear containment structures, the specimens are heavily reinforced in the longitudinal direction (*x*-direction) and have some reinforced in the transverse shear direction (*z*-direction). The height of the members *h*, (610 mm) and effective depth, d_v, (508 mm) is representative of containment walls at a scale of approximately two-to-one. The specimens were designed to be representative of shell elements being loaded in one out-of-plane direction, such members are commonly referred to as a 'shell strips'. The advantage of testing shell strips, rather than shell elements with out-of-plane shear in multiple directions, is that it enables the use of Digital Image Correlation (DIC) equipment to measure the entire displacement field on the surface of the elements throughout loading. The specimen details are provided in Figure 6. The specimens were constructed and tested to failure at the Constructed Facilities Lab at North Carolina State University. It should be noted that for clarity US units are shown in the drawings as this was the unit system used to construct the members (a conversion is provided in the caption).

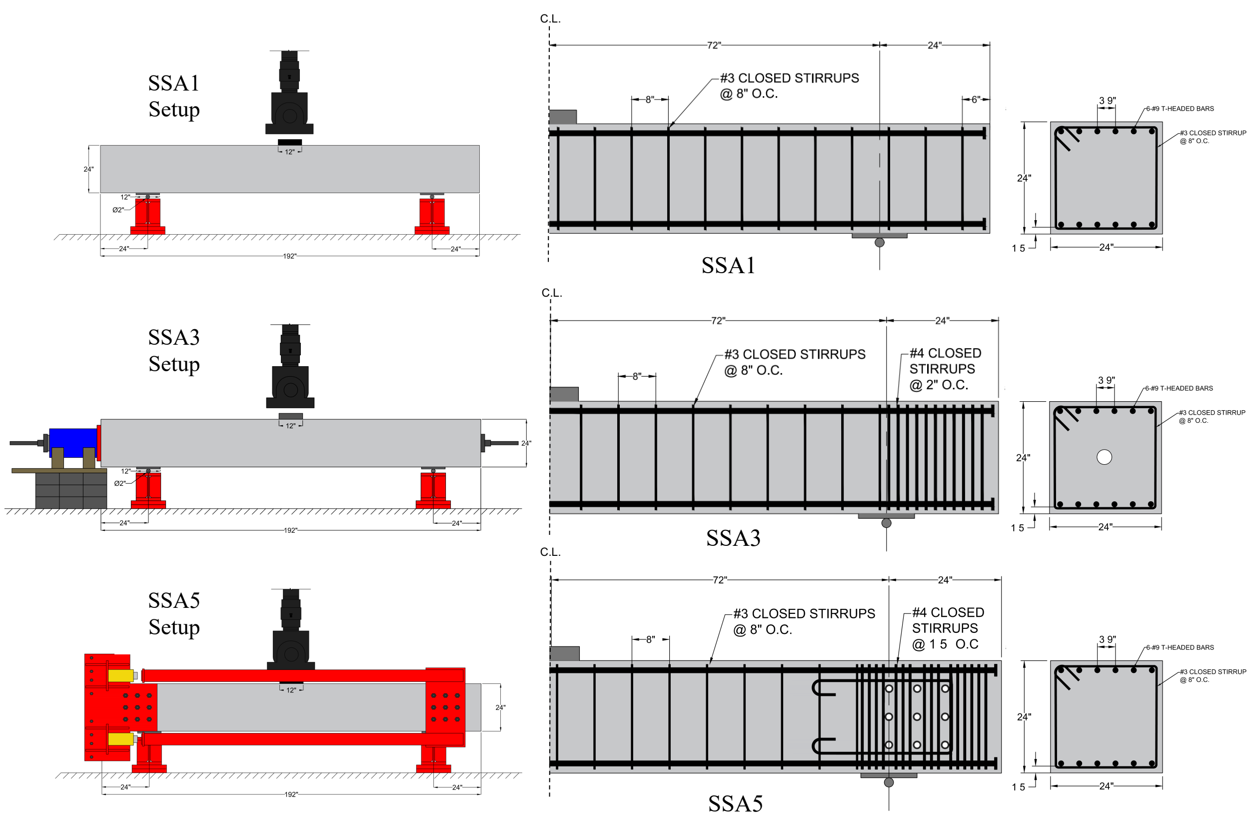


Figure 6. Experimental Setup (left) and specimen details (right) [25.4 mm = 1 in.].

For the longitudinal, *x*-direction reinforcement, all three specimens had six #9 HRC 555 T-headed bars in a single row, top and bottom. This provided a reinforcement ratio, ρ_x, of 2.08%. The transverse reinforcement was Grade 60 #3 closed stirrups spaced nominally at 203 mm (8 in.). The transverse reinforcement in the *z*-direction, resulted in a reinforcement ratio, ρ_z, of 0.115%. The average test day concrete compressive strength ranged from 30.3 MPa to 31.2 MPa. While the properties of SSA1, SSA3 and SSA5 were nominally identical, the loading configurations were varied. SSA1 consisted only of out-of-plane loading, SSA3 consisted of out-of-plane loading combined with compression and SSA5 consisted of out of plane loading combined with tension. For the tension loading a reaction system was mounted on the specimen and for the compression loading the compression was applied through a bar in the centre of the specimen. Figure 4 shows the experimental setup used for each specimen. A summary of the specimen properties, loading ratios and peak loads achieved are provided in Table 1. More information on the experimental program and specimen details can be found elsewhere (Stuart, 2021).

Table 1. Specimen properties, experimental observations and 3D SEM predictions.

Name	f'_c (MPa)	ρ_x (%)	f_{yx} (MPa)	ρ_z (%)	f_{yz} (MPa)	$\rho_z f_{yz}$ (MPa)	$f_x{:}v_{xz}$ †	$N_x{:}V_{xz}$ ‡	V_{xz} (kN)	v_{xz} (MPa)	$\frac{v_{xz\ exp}}{v_{xz\ 3D\ SEM}}$ (d_v)	$\frac{v_{xz\ exp}}{v_{xz\ 3D\ SEM}}$ (h)
SSA1	30.3	2.08	601	0.115	457	0.526	0:1	0:1	485	1.58	0.91	0.97
SSA3	30.9	2.08	601	0.115	457	0.526	-1.36:1	-2:1	708	2.31	1.15	1.19
SSA5	31.2	2.08	601	0.115	457	0.526	0.80:1	1:1	463	1.51	0.92	0.94

† Ratio of axial stress to shear stress at peak load.
‡ Applied axial force to shear force ratio.

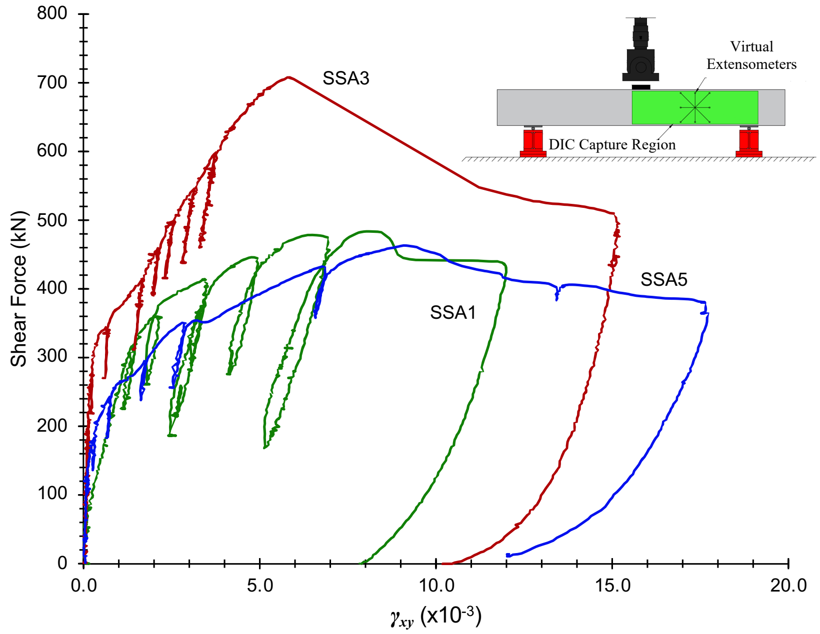


Figure 7. Experimentally observed load deformation response for SSA1, SSA3 and SSA5.

One of the goals of the experimental program was to compare the response of shell strip elements subjected to axial tension versus axial compression. Figure 7 provides a summary of the average shear strain determined in the shear span versus the shear force throughout loading. The average shear strains are determined in the shear span by using the DIC data to obtain one axial strain in the *x*-direction, one axial strain in the transverse, *z*-direction, and two diagonal strains at +/- 45 degrees to the *x*-axis. The rosette pattern used to measure the average shear strains is shown in Figure 7. As seen in the figure, SSA3 which was subjected to out-of-plane shear combined with axial compression, exhibited the stiffest post-cracking response and reached the highest peak load. SSA1, which was subjected to out-of-plane shear combined with tension exhibited the lowest post-cracking stiffness and reached the lowest peak shear force. SSA1 had no axial stress applied and had a shear strength between SSA3 and SSA5.

In addition to determining the average strain response, the DIC data can be used to show detailed information on how the principal tensile strains vary in the shear span. The principal tensile strain plots for SSA1, SSA3 and SSA5 are shown in Figure 8 for the specimens at peak load. The principal tensile strains are also indicative of the crack patterns in the members and can be used to evaluate where cracking is occurring. Reporting these values is important for the assessment of shell structures, for comparison with nonlinear finite element techniques and to inform learning algorithms and digital twin approaches that require cracking information throughout loading.

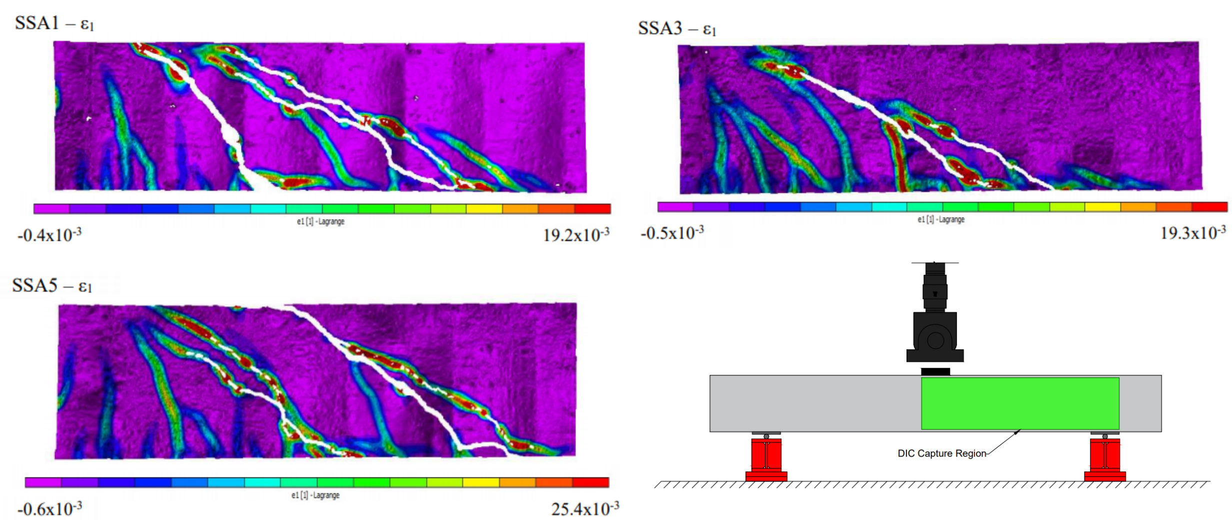


Figure 8. Principal tensile strains measured by DIC at peak load for specimens SSA1, SSA3 and SSA5.

COMPARISON OF SHELL STRIP EXPERIMENTS WITH 3D SEM PREDICTIONS

A summary of the test-to-predicted ratios for the three experiments is shown in Table 1 for when an effective height of d_v and h are used in the analyses. When d_v is used as the effective height, the test-to-predicted ratios range from 0.91 to 1.15 and when an effective height of h is used, the test-to-predicted ratios range from 0.94 to 1.19. In both cases these results indicate that the 3D SEM is capable of predicting the peak load of the shear critical shell strip experiments conducted.

The 3D SEM is not only capable of predicting the peak load of shell elements subjected to shear but is also capable of predicting the full load deformation response of the elements. Figure 9 shows the experimental and predicted shear stress versus shear strain response for SSA1, SSA3 and SSA5. The figures show the 3D SEM predictions for when both an effective height of d_v and h are used in the analyses. As previously discussed, in all three cases the peak load is reasonably well predicted. Additionally, the post-cracking stiffness, also known as the cracked elastic stiffness, is well predicted. This is particularly impressive given that the analysis is only using a single 3D triaxial MCFT element to make the predictions. The ability for the method to reasonably predict the cracked elastic stiffness is also useful in a variety of scenarios.

Specifically, having the ability to accurately predict the cracked elastic stiffness of a member in a rapid and simplified manner can be useful to inform more complex finite element models. For example, when developing finite element models of building or building components in nuclear facilities the 3D SEM can be used to inform the stiffness assumptions or detailed constitutive input parameters used in the analysis. Utilizing the correct stiffness of members in structural models is important to predict cracks at minor or moderate damage limit states, correctly capturing the dynamic properties of structures when subjected to seismicity and in correctly accounting for damping characteristics in the structures. Of additional importance is being able to establish when the elements transition from elastic response to cracked response and from minor damage to moderate or severe damage. As indicated in Figure 9 the 3D SEM is able to provide reasonable results in this regard.

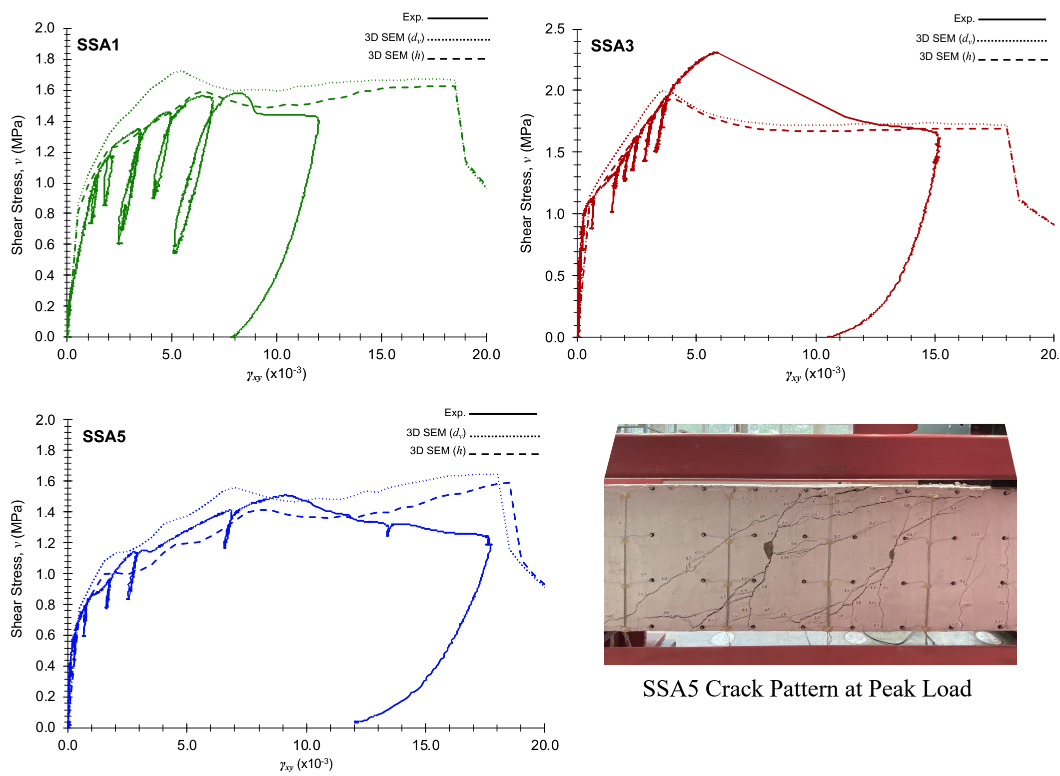


Figure 9. Shear stress versus shear strain response from experiments and 3D SEM predictions.

CONCLUSIONS

This paper presented a new methodology to predict the shear response of shell elements subjected to the eight stress resultants called the 3D SEM. The 3D SEM uses the sectional properties of the shell elements, the applied loading conditions and a single three-dimensional MCFT element to predict the sectional shear response of the shell elements throughout loading. The 3D SEM methodology was compared to the SP series of shell element experiments conducted by Adebar and Collins (1994) to examine the ability of the method to predict the combined effects of in-plane and out-of-plane shear. The results of the 3D SEM provided reasonable results of the interaction between in-plane and out-of-plane shear including the asymmetry that occurs as a result of the in-plane and out-of-plane compression fields interacting.

To further examine, verify and validate the 3D SEM tool and to explore the influence of axial stresses on out-of-plane shear response of containment structures, three 4877 x 610 x 610 mm reinforced concrete test specimens were constructed and tested to failure. The shell strip experiments were constructed to have quantities of longitudinal reinforcement typical of containment wall structures. All members also contained transverse, out-of-plane, shear reinforcement. The response of the members was monitored with full field of view DIC equipment. The results of the experiment compared the influence of axial tension and axial compression on the shear response of reinforced concrete shell strips. The results indicate that the cracked elastic stiffness and peak shear strength of members with axial compression was higher than in members without axial compression. Conversely, axial tensile stresses reduced the cracked elastic stiffness and peak strength of the members. The 3D SEM was used to predict the peak load of the experiments and the full load deformation response of the members. It was demonstrated that the 3D SEM is able to predict the response of the experiments and capture the differences in response of members subjected to combined out-of-plane shear and axial stresses.

In addition to examining the average shear stress-shear strain response, DIC was used to capture the strain maps at the peak load. The full field of view DIC data captured the member cracking and displacement field of the members. As field measurement techniques improve using such approaches in conjunction with simplified analysis techniques, such as the 3D SEM, could be used to inform structural assessments, structural health monitoring approaches and digital twin technologies in the nuclear industry.

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