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# A METHOD FOR SCALING IN-STRUCTURE RESPONSE SPECTRA

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## ABSTRACT

Sometimes, during the life of a nuclear power plant, it is necessary to assess the safety margins inherent in the plant seismic design. Part of that assessment is the re-evaluation of the plant equipment and systems for new in-structure response spectra generated from a seismic input ground motion, which is different to the one used in their original design.

Re-doing the plant seismic analyses can present major economic and labor challenges. To avoid performing a new seismic analysis of the structures, several methods are used to scale the existing in-structure response spectra to generate approximate in-structure response spectra consistent with the new input ground motion. These methods range from simply scaling the in-structure response spectra by the ratio of the original and new input spectra at the dominant frequency of the structures, EPRI (1991), to more elaborated methods based on direct generation of in-structure response spectra, Asfura et al. (1990).

This paper presents a new method based on Random Vibration Theory (RVT) to scale existing in-structure response spectra to estimate in-structure response spectra for a new input motion. Using RVT techniques, the method starts by converting the existing in-structure response spectra and the original input ground response spectrum into equivalent power spectral density (PSD) functions. The ratio of the PSD of a particular in-structure response spectrum to the PSD of the original input ground response spectrum can be interpreted as the transfer function for that particular response. If the transfer function does not change with the new input, this transfer function and the PSD function of the new input ground response spectrum can be used to calculate the PSD function of the in-structure response spectrum for the new input motion. Finally, RVT techniques can be used again to convert this PSD function into the scaled in-structure response spectrum.

This approach results in accurate scaled in-structure response spectra. The method is easy to implement and automatize, reducing the cost and the labor effort needed to develop approximate in-structure response spectra consistent with a new input ground motion.

The method presented in this paper is similar to the approach used in soil-structure interaction (SSI) analysis to calculate new in-structure responses for a new input ground motion when structural and soil parameters do not change with the new input. There, the Fourier transforms are used instead of the PSD functions.

## METHODOLOGY

The standard practice in seismic analysis of structures considers that the input ground motions in the three orthogonal directions are uncorrelated. Then, the PSD function of a structural response in a certain direction x at frequency ω can be expressed as follows, Bendat and Piersol (1980):

$$S_{ox}(\omega) = |H_{xx}(\omega)|^2 S_x(\omega) + |H_{xy}(\omega)|^2 S_y(\omega) + |H_{xz}(\omega)|^2 S_z(\omega) \quad (1)$$

In the equation above, $S_{ox}(\omega)$ is the PSD function of a response in the x direction, $H_{xj}(\omega)$ is the frequency response function in the x direction due to the input in the j direction (j = x, y, z), and $S_j(\omega)$ is the PSD function of the input in the j direction.

If x and y are horizontal directions, standard practice considers that $S_x(\omega)$ and $S_y(\omega)$ are equal. If the effect of rocking in the structural responses is small, or in case that the vertical input is proportional to the horizontal input, Equation 1 for the horizontal response can be rewritten as:

$$S_{oh}(\omega) = TF_h(\omega) S_h(\omega) \quad (2)$$

In Equation 2, $S_{oh}(\omega)$ is the PSD function of a response in the horizontal direction, $TF_h(\omega)$ is the transfer function, and $S_h(\omega)$ is the PSD function of the horizontal input ground motion. A similar equation can be written for the vertical response so, for the rest of the paper, the subscript h will be eliminated.

If a structure is analyzed for two different input ground motions ("old" and "new"), and the transfer functions remain constant, the following relationship is valid at each frequency ω:

$$S_{o,new}(\omega) = S_{o,old}(\omega)[S_{new}(\omega)/S_{old}(\omega)] \quad (3)$$

In Equation 3, $S_{o,new}(\omega)$ is the PSD function of the response of the new analysis at frequency ω, $S_{o,old}(\omega)$ is the PSD function of the response of the old (existing) analysis at frequency ω, $S_{new}(\omega)$ is the PSD function of the input ground motion for the new analysis at frequency ω, and $S_{old}(\omega)$ is the PSD function of the input ground motion for the old analysis at frequency ω.

Thus, since the results for the old analysis and the input ground motions for the old and new analyses are known, the PSD function of the results for the new analysis can be calculated directly from the relationship in Equation 3, without performing the second analysis.

Equation 3 gives the relationship between the PSD functions of two input motions and the responses to those two analyses. This relationship is used to develop a method to scale existing in-structure response spectra from a given input ground motion to obtain the in-structure response spectra from a different input ground motion.

In practice, only the in-structure response spectra of the old analysis, and the response spectra of the old and new input ground motions are known. Therefore, to use Equation 3 to scale existing in-structure response spectra is necessary to convert the known response spectra into their equivalent PSD functions. This can be done using RVT techniques, Der Kiureghian (1980, 1981) and Igusa and Der Kiureghian (1983).

The new scaling method presented in this paper begins with the conversion of both, old and new, input response spectra and the old in-structure response spectra into their PSD functions. Next, the PSD function of the old in-structure response spectra is scaled by the ratio of the PSD functions of the new and old input ground motions to obtain the PSD function of the new in-structure response spectra. Finally, this new PSD function is converted into the in-structure response spectra associated to the new input ground motion.

This new scaling method might be simplified by using the approximate relationship shown in Equation 4.

$$[S_{new}(\omega)/S_{old}(\omega)] \sim [GMS_{new}(\omega)/GMS_{old}(\omega)]^2 \quad (4)$$

In Equation 4, $GMS_{new}(\omega)$ is the response spectrum of the new input ground motion, and $GMS_{old}(\omega)$ is the response spectrum of the old input ground motion. This relationship is true for zero damping, but as shown in the next section, it is still a good approximation for low damping values spectra and for PSD functions generated from smooth input response spectra. Thus, Equation 3 might be rewritten as:

$$S_{o,new}(\omega) = S_{o,old}(\omega)[GMS_{new}(\omega)/GMS_{old}(\omega)]^2 \quad (5)$$

Equation 5 simplifies this new method, since only the old in-structure response spectra need to be converted into PSD functions. In the next section, this new scaling method is applied to two different structures and two different input ground motions. In those examples, Equations 3 and 5 are both used, and their results compared.

## SPECTRA SCALING EXAMPLES

To demonstrate this new scaling method, two typical structures, as those found in nuclear power plants, were analyzed for two input ground motions defined by two "design" response spectra. First, SSI analyses were performed for these two structures with two acceleration time histories with response spectra matching the "design" input ground response spectra. Then, the in-structure response spectra calculated with the second SSI analysis were scaled using the method presented in this paper and compared to the in-structure response spectra calculated with the first SSI analysis.

### *Structural Models*

The first structure is a typical Reactor Building in a nuclear power plant. This building is founded in a soil profile with an approximate depth of 475 feet and with shear wave velocities ranging from about 1,200 ft/sec at the surface to about 7,000 ft/sec at the bottom of the profile. The building is embedded about 45 feet and resting on a soil layer with shear wave velocity of about 2,800 ft/sec. The second structure is a smaller building but with a more irregular seismic response. This building is founded in a soil profile with an approximate depth of 475 feet and with shear wave velocities ranging from about 770 ft/sec at the surface to about 5,000 ft/sec at the bottom of the profile. The building is embedded about 20 feet and resting on a soil layer with shear wave velocity of about 1,000 ft/sec. The finite element models of these structures are shown in Figure 1. The dominant frequency for both buildings, including the soil, is close to 5 hz.

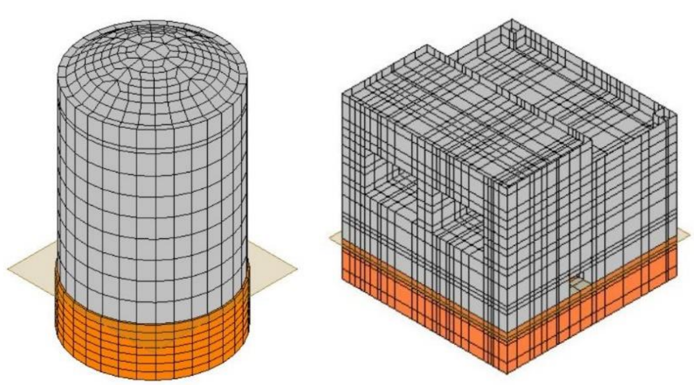

Figure 1. Building 1: Left Model; Building 2: Right Model

### *Input Ground Motions*

Two different input ground motions were used in the examples of the in-structure spectra scaling method. As a first step, two input response spectra were selected. These spectra are shown in Figure 2 in solid lines, and they will be referred to as "design" spectra in the rest of this paper. The highest spectrum (blue line) in Figure 2 will be referred to as Input 1 (new input) and the lower spectrum (red line) as Input 2 (old input).

As part of the verification of the scaling method, two acceleration time histories were developed such that their response spectra matched the design spectra. In Figure 2, the design spectra (solid lines) and the spectra from the time histories (dotted lines) are shown together to demonstrate the matching of the time histories spectra.

Using RVT techniques, PSD functions were developed directly from the two design input response spectra. These PSD functions are also shown in Figure 2. The spikes observed in the PSD function plots are due to corner points in the design input response spectra.

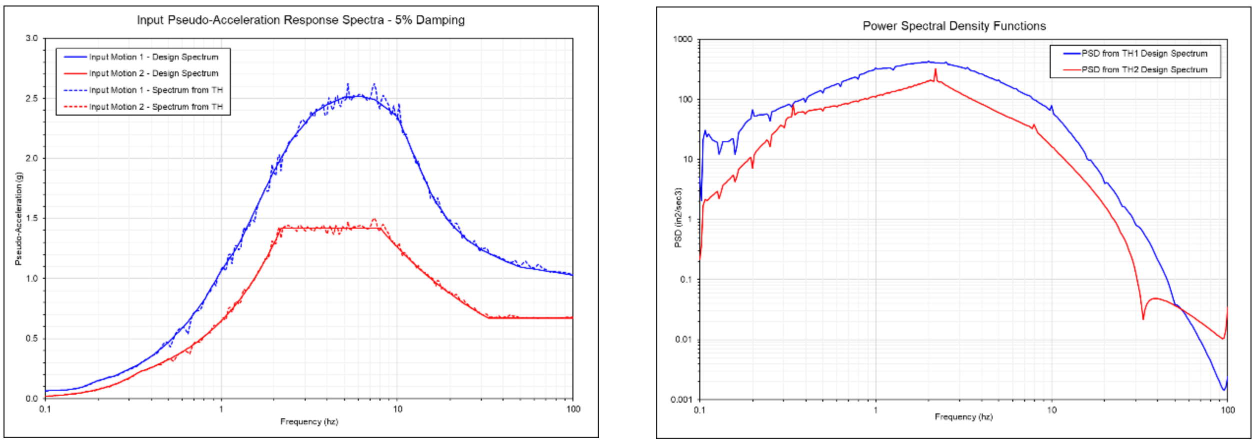


Figure 2. Input Ground Motions: Response Spectra and PSD Functions

The ratio of the PSD functions $[S_1(\omega)/S_2(\omega)]$ and the ratio of the design input response spectra (5% damping) square $[GMS_1(\omega)/GMS_2(\omega)]^2$ are shown in Figure 3. Both ratios are similar in the frequency range containing most of the energy of the input motions. The large peak in the ratio of the PSD functions seen in Figure 3 at about 33 hz. is due to the spurious valley in the PSD function of Input 2 at that frequency.

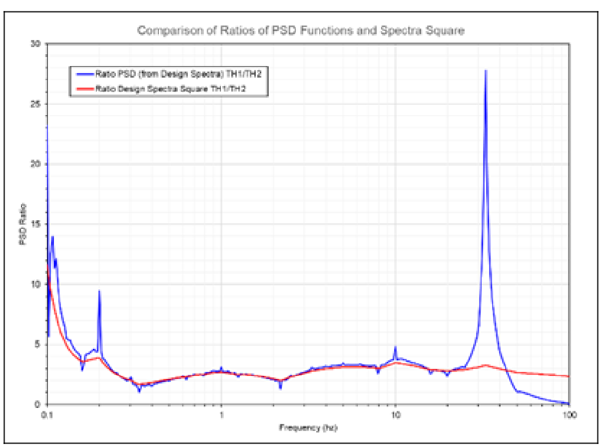


Figure 3. PSD Functions and Spectra Square Ratios

***Soil-Structure Interaction Analyses***

To have an old and new (target) set of in-structure response spectra, time history SSI analyses were performed for the two structures described before using both time histories defined in the previous sub-section.

From these SSI analyses, in-structure response spectra were calculated at four locations in each structure. For Building 1 three locations were selected at the internal structure, nodes 1 to 3, and one at the top of the containment, node 4. Node 1 is about 10 feet above ground level, nodes 2 and 3 are about 65 feet above ground level, and node 4 is about 170 feet above ground level. For Building 2, node 1 is about 10 feet above ground level, node 2 about 25 feet above ground level, node 3 about 50 feet above ground level, and node 4 about 60 feet above ground level.

Figure 4 shows the in-structure response spectra at Buildings 1 and 2 for input motion 1 ("new" analysis), and Figure 5 shows the in-structure response spectra at Buildings 1 and 2 for input motion 2 ("old" analysis).

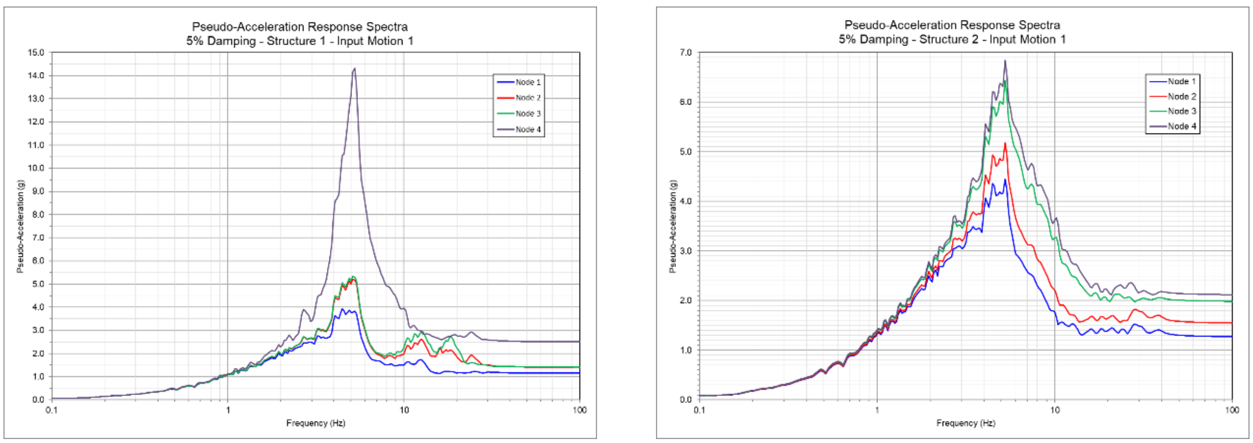


Figure 4. In-structure Pseudo-acceleration Response Spectra from SSI Analyses. Input Motion 1

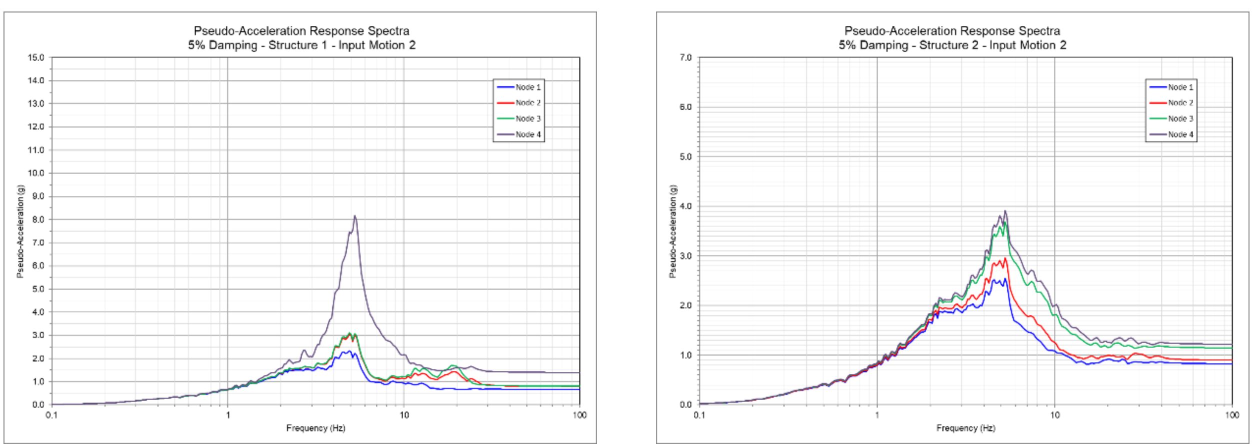


Figure 5. In-structure Pseudo-acceleration Response Spectra from SSI Analyses. Input Motion 2

***In-Structure Response Spectra Scaling***

To demonstrate the new scaling methodology presented in this paper, the in-structure response spectra shown in Figure 5 ("old" analysis) will be scaled and compare to the in-structure response spectra shown in Figure 4 ("new" analysis).

The process starts by converting the in-structure response spectra in Figure 5 ("old" analysis) into their equivalent PSD functions. Next, these PSD functions are scaled by the ratio of the PSD functions of the input motions (Equation 3, blue line in Figure 3) or alternatively, by the ratio square of the input response spectra (Equation 5, red line in Figure 3). The scaled values are the PSD function values of the in-structure response spectra of the "new" analysis. To finalize the process, these PSD functions are converted to "scaled" in-structure response spectra using RVT techniques.

Figure 6 shows, for Building 1, the comparison of the scaled in-structure response spectra (red and green lines) to the "exact" in-structure response spectra calculated by time-history SSI analysis (blue line). Figure 7 shows the same comparison for Building 2. These results demonstrate that the methodology presented in this paper accurately estimates the in-structure response spectra for a new input motion using old in-structure response spectra from a different input motion. They also demonstrate that using the ratio of the square of the input response spectra (Equation 5) is equivalent to using the ratio of the PSD functions of the input motions (Equation 3) in the calculation of the PSD functions of the new in-structure response spectra.

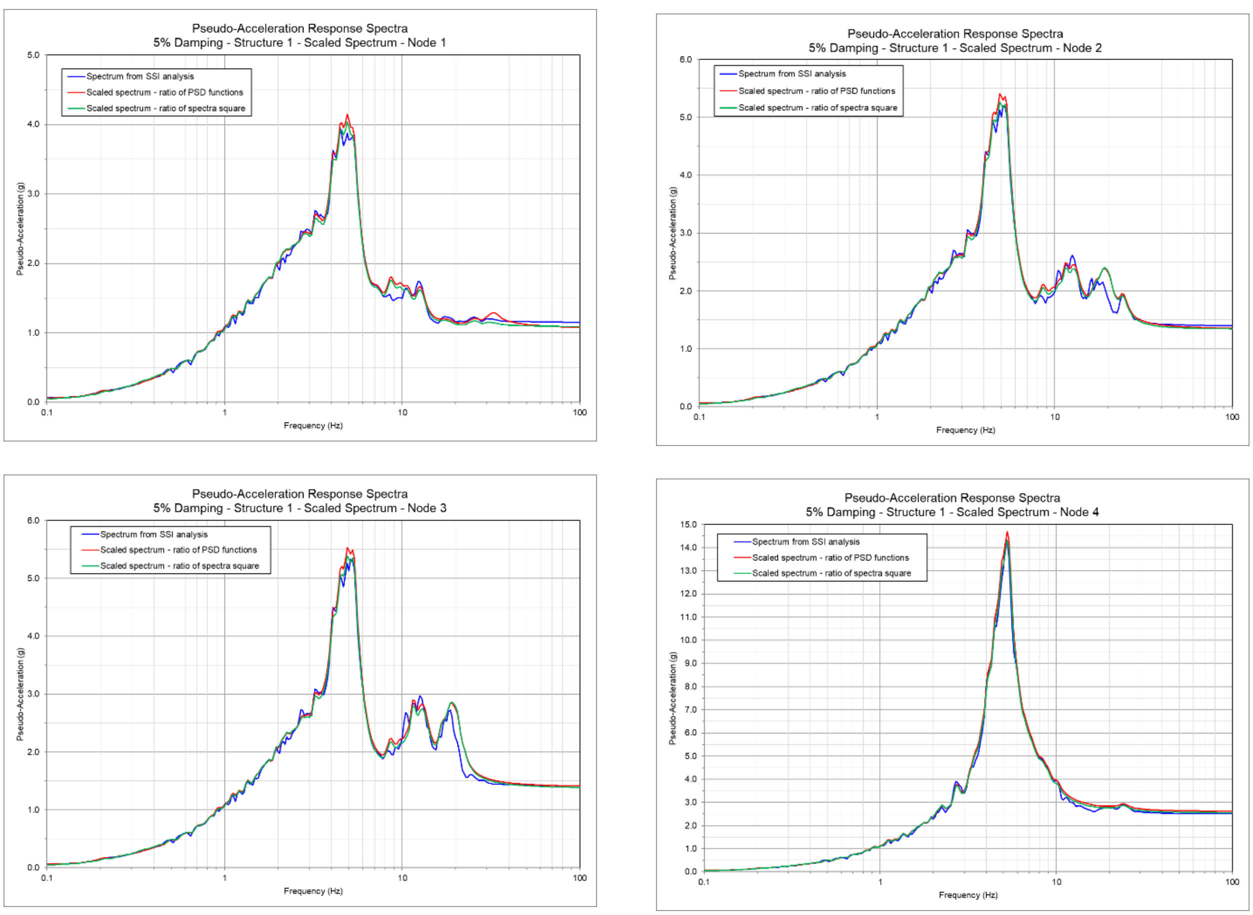


Figure 6. Comparison of Scaled and Exact In-structure Response Spectra. Building 1

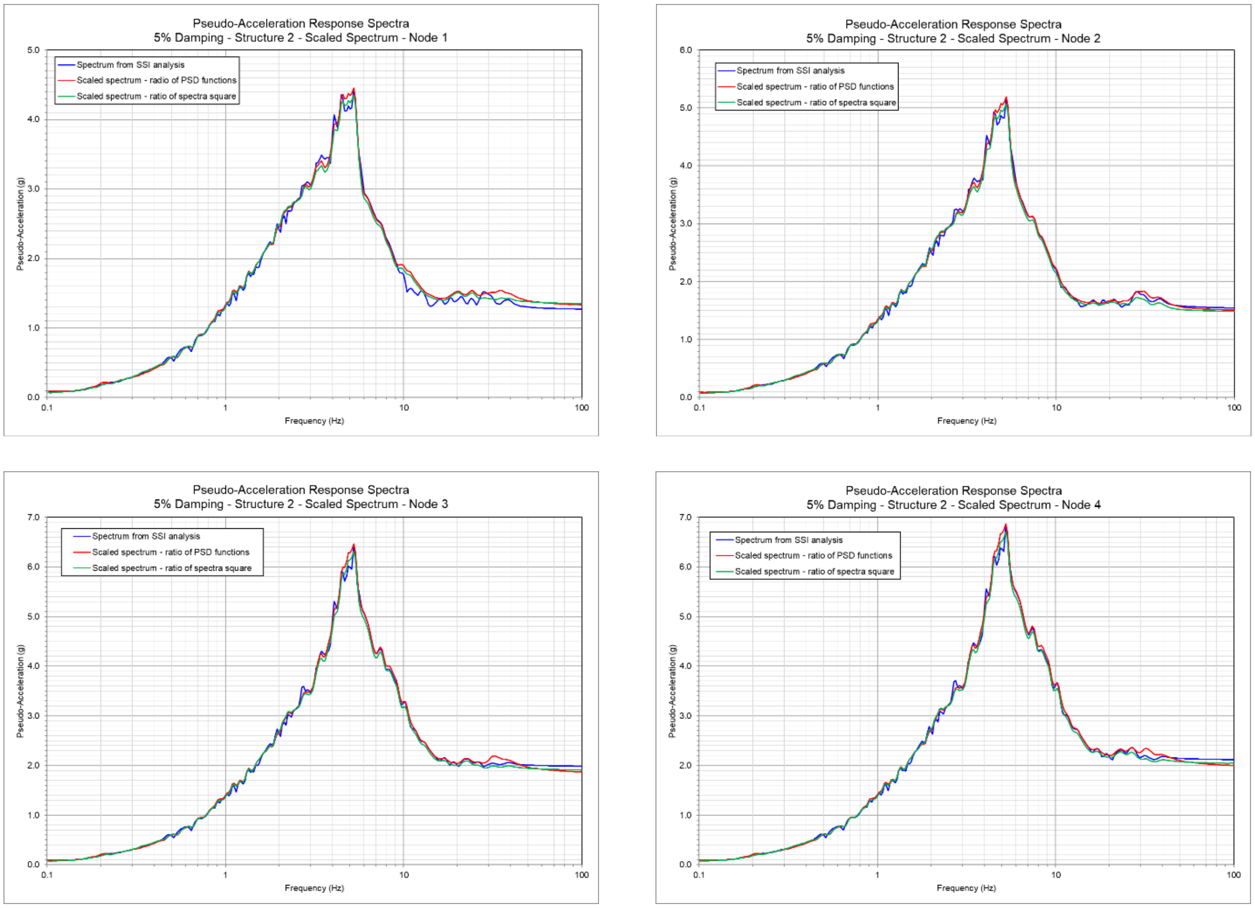


Figure 7. Comparison of Scaled and Exact In-structure Response Spectra. Building 2

## CONCLUSION

The new scaling method presented in this paper was applied to structures and input motions like those found in actual practice. The results of the examples demonstrate that the method accurately estimates in-structure response spectra associated to a new input motion from the results of an old analysis. These examples also demonstrate that, in the calculation of the PSD functions of the new in-structure response spectra, using the ratio of the square of the input response spectra (Equation 5) or using the ratio of the PSD functions of the input motions (Equation 3) gives similar results.

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