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BAYESIAN META-MODEL (MOCABA) OF FUEL ASSEMBLY SPACER GRID DEFORMATIONS FOR USE IN SEISMIC FRAGILITY ANALYSIS

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ABSTRACT

A multivariate Bayesian meta-model of the seismic-induced demand is developed, which is based on the generalized MOCABA framework, see Hoefer & Buss (2021). The MOCABA framework was originally developed for application to functions of nuclear data (e.g. reactor power distribution).

The seismic demand experienced by a component is the basis for quantifying the fragility of a component, i.e. the probability of the demand exceeding the capacity. In the present study the components of interest are fuel assemblies of a PWR. The seismic demand is characterized by the permanent deformations of fuel assembly spacer grids. The seismic loading is characterized by a set of 13 seismic intensity measures, including the peak ground acceleration (PGA), Arias intensity, cumulative absolute velocity, strong motion duration, and spectral accelerations (as well as integrals thereof, see Figure 1), which are the input parameters. The dataset for training and testing of the meta-model is based on 180 ground motion time histories generated within the EURATOM R&D program "NARSIS", covering a wide range of seismic intensity, see NARSIS (2019).

In the generalized MOCABA framework the user can choose from different multivariate probability distribution models. The present study is focusing on distribution models involving log-normal, log-Johnson and log-empirical distributions (see Hoefer & Buss (2021) for details). So far, the MOCABA meta-model has been applied to a censored dataset of size 116, where the samples leading to zero permanent deformation are screened out. The demand distributions predicted by the meta-model are, therefore, conditional on the assumption of non-zero deformation.

INTRODUCTION

Row models of fuel assemblies are widely used to demonstrate adequate performance of fuel assemblies in light water reactors, as requested in the context of seismic design or beyond-design safety evaluations. The main purpose of the model is to capture the seismic loading experienced by fuel assemblies in the form of impacts, either with neighboring fuel assemblies or - in case of fuel assemblies along the edge of the core - with the core baffle. For the safety demonstration, the key output quantity is the predicted **permanent deformation** of the spacer grids. More specifically the deformation is limited to levels for which a disturbance of the control rod insertion in case of a reactor trip is excluded.

The computational cost of the non-linear time-history analyses with the row models is inherently high. On the other hand, a large number of analyses is needed in the context of seismic safety. Firstly, analyses should be performed for various core configurations, accounting for different fuel assemblies and

spacer grid properties. Secondly, fragility curves used for SMA and seismic PSA cover a wide range of ground motion levels. For components analyzed by linear models, the extrapolation to higher ground motion levels – as utilized in the standard fragility analysis based on the scaling factor concept - can be justified. However, for non-linear models the availability of analysis results for various ground motion levels is definitely preferable. Thirdly, the effects of seismic ground motions on mechanical components does not depend exclusively on a single ground motion parameter, such as the peak ground acceleration (PGA) that is commonly used as the independent variable in standard fragility analysis. Other seismic intensity measures (IM) can be equally or even stronger correlated with seismic-induced damage.

Under these circumstances, it makes sense to scrutinize alternative meta-models for their **potential benefits** and **limitations** in supporting safety studies. Meta-models capture the relationship between input variables (\rightarrow seismic intensity measures) and output variables (\rightarrow permanent spacer grid deformation). In the present paper, a **multivariate Bayesian** meta-model is used, which is based on the so-called generalized MOCABA framework, see Hoefer & Buss (2021). The MOCABA framework was originally developed for application to functions of nuclear data (e.g. reactor power distribution).

DESCRIPTION OF THE DATASET

The present section describes the input and output data forming the dataset used to train and test the MOCABA meta-model. Similar datasets have been used in Altieri et. al. (2020) and Pellissetti et.al. (2021).

Input data

The **raw** input data are represented by 180 ground motion time histories, grouped in six batches of 30 time histories. The corresponding response spectra are shown in Figure 1 (left part). Each batch corresponds to a seismic intensity range, as suggested by the different colors. Regarding the procedure for the generation of the ground motion time histories it is referred to NARSIS (2019).



Figure 1. Left: Ground motion response spectra for direction X, D=5% - all 180 accelerograms; Right: Fuel assembly row model

The time histories have been processed to produce seismic intensity measures (IM), as compiled in Table 1 below. To begin with, **distinct** IMs are obtained for each directional component (two horizontal and one vertical). The **final input data** for the meta-model are then obtained by taking, for each IM, the **geometric mean** of the corresponding IM for the **two horizontal directions**.

1	PGA	peak ground acceleration
2	PGV	peak ground velocity
3	PGD	peak ground displacement
4	I _A	Arias intensity
5	CAV	cumulative absolute velocity
6	SA ₁	spectral acceleration at 1.3 Hz (close to the first frequency of the FA)
7	SA _{1,avg}	average of the spectral acceleration between 1.3 and 2.4 Hz
8	SVr ₁	spectral relative velocity at 1.3 Hz
9	SVr _{1,avg}	average of the spectral relative velocity between 1.3 and 2.4 Hz
10	Av	ratio PGA/PGV
11	SA ₃	spectral acceleration at 4.9 Hz (close to the third frequency of the FA)
12	T _{SM,F}	duration of the strong motion (French definition, T95-T5)
13	T _{SM,US}	duration of the strong motion (US definition, T ₇₅ -T ₅)

Table 1: IM considered in the present study

Output data

Each of the 180 sets of ground motion time histories is propagated a.) through a building model and b.) through a fuel assembly row model as shown in Figure 1 (right part). The analysis is simplified, in the sense that the building time histories at the reactor pressure vessel support level are applied directly to the row model, without considering the dynamic behavior of the RPV internals. The fuel assembly row model is similar to the one presented in Pellissetti et.al. (2017). See also NARSIS (2019).

The fuel assembly row model predicts the permanent spacer grid deformation for each of the eight spacer grids (\rightarrow springs in Figure 1) along each of the fuel assemblies (\rightarrow beam elements in Figure 1). The full data set includes – for each time history – a total of 64 spacer grid deformation data: 8 spacer grids per fuel assembly, 2 directions, 2 fuel assemblies (the ones at the edge of the row, next to the core shroud), 2 different row models (one with 13 fuel assemblies, the other one with 17). The data of the present study refer to spacer grid **5** (from the top; the spacer grids 4 and 5 experience the strongest impact and hence the largest deformations), **arithmetic average of directions** X and Y, fuel assembly **left, along the baffle**, row model with **17** fuel assemblies. This location is considered, since it can be equipped with a rod cluster control assembly, and can experience significant grid deformation.

DESCRIPTION OF THE BAYESIAN META MODEL

Mathematical framework

The Bayesian meta-model considered in this paper is based on the generalized MOCABA framework described in Hoefer & Buss (2021). Its objective is to **map input** parameter values (e.g., the values of the 13 seismic intensity measures in Table 1) onto the corresponding values of **related response variables** (e.g., variables describing permanent spacer grid deformations of fuel assemblies in a reactor core). Using the same notation as in Hoefer & Buss (2021), the **response variables** are identified with the components of a vector \mathbf{y}_A , and the **input parameters** are identified with the components of a vector \mathbf{y}_B . Collecting \mathbf{y}_A and \mathbf{y}_B in a combined vector $\mathbf{y} = (\mathbf{y}_A^T, \mathbf{y}_B^T)^T = (y_1, \dots, y_n)^T$, the prior probability density function (pdf) $p(\mathbf{y})$ of \mathbf{y} is parameterized as follows:

$$p(\mathbf{y}) \propto \exp\left(-\frac{Q_0}{2}\right) \left| \prod_{i=1}^n \frac{\partial f_i(y_i)}{\partial y_i} \right|, \quad Q_0 = (\mathbf{f}(\mathbf{y}) - \mathbf{z}_0)^T \mathbf{\Sigma}_0^{-1} (\mathbf{f}(\mathbf{y}) - \mathbf{z}_0)$$
(1)

Here, f represents a vector of invertible variable transformations that maps the components of the vector y onto the components of a transformed vector z:

$$\mathbf{z} = (\mathbf{z}_{A}^{T}, \mathbf{z}_{B}^{T})^{T} = (z_{1}, ..., z_{n})^{T} = \mathbf{f}(\mathbf{y}) = (f_{1}(y_{1}), ..., f_{n}(y_{n}))^{T}$$

$$\mathbf{y} = (\mathbf{y}_{A}^{T}, \mathbf{y}_{B}^{T})^{T} = (y_{1}, ..., y_{n})^{T} = \mathbf{f}^{-1}(\mathbf{z}) = (f_{1}^{-1}(z_{1}), ..., f_{n}^{-1}(z_{n}))^{T}$$
(2)

Within this framework, the transformed vector z follows a **multivariate normal** distribution, defined by the mean vector z_0 and the covariance matrix Σ_0 . Hence, y is described by a **distribution model** defined by the mean vector z_0 , the covariance matrix Σ_0 , and by the **transformation parameters** defining the transformation vector f. Since the physical input parameters and response variables considered in this paper (represented by the components y_i of y) can only assume **positive** values, we consider here three different types of **left-bounded** distribution models for the components y_i , which are denoted as:

- log-normal distribution model
- log-empirical distribution model
- log-Johnson distribution model

These distribution models correspond to the following parameterizations of the transformations f_i :

• log – normal distribution:
$$z_i = f_i(y_i) = \ln(y_i - y_i^L)$$
 (3)

• log – empirical distribution:
$$z_i = f_i(y_i) = F_{N,i}^{-1} \left(F_{E,i} \left(\ln(y_i - y_i^L) \right) \right)$$
 (4)

• log – Johnson distribution:
$$z_i = f_i(y_i) = \sinh^{-1}\left(\frac{\ln(y_i - y_i^L) - a_i}{b_i}\right)$$
 (5)

Using either of these three distribution models, the domain of the pdf $p(y_i)$ of y_i is given by $y_i \ge y_i^L$. For the analysis presented **in this paper**, a lower bound value of $y_i^L = 0$ is used. In the definition of the log-empirical distribution model, $F_{N,i}$ denotes the distribution function of the normally distributed variable z_i , and $F_{E,i}$ denotes the empirical distribution function of $\ln(y_i - y_i^L)$.

In the **training phase**, the distribution model parameters of the prior pdf p(y) are estimated from a database $Y = \{y_1, ..., y_m\}$ of *m* observations of *y*. Using the log-empirical or log-Johnson distribution model, **first the transformation parameters** of f_i are estimated from the data set *Y* for each component *i* separately. For the log-empirical distribution model, the empirical distribution function $F_{E,i}$ is estimated from the **order statistics** of $\ln(y_i - y_i^L)$ observations. When using a log-Johnson distribution model, the model parameters a_i and b_i are fitted by **maximizing the corresponding log-likelihood** function (see Hoefer & Buss (2021)).

Having estimated the transformations f_i , they are used to **map** the database Y of observations of y onto a database $Z = \{z_1, ..., z_m\}$ of observations of z. Since the transformations f_i are chosen such that z is approximately multivariate normally distributed, the mean vector z_0 and the covariance matrix Σ_0

defining the multivariate normal pdf of z, are estimated by applying the corresponding **unbiased** estimators to the data set Z:

$$\mathbf{z}_{0} = \begin{pmatrix} \mathbf{z}_{0A} \\ \mathbf{z}_{0B} \end{pmatrix} = \frac{1}{m} \sum_{j=1}^{m} \mathbf{z}_{j}, \quad \mathbf{\Sigma}_{0} = \begin{pmatrix} \mathbf{\Sigma}_{0A} & \mathbf{\Sigma}_{0AB} \\ \mathbf{\Sigma}_{0AB}^{T} & \mathbf{\Sigma}_{0B} \end{pmatrix} = \frac{1}{m-1} \sum_{j=1}^{m} (\mathbf{z}_{j} - \mathbf{z}_{0}) (\mathbf{z}_{j} - \mathbf{z}_{0})^{T}. \tag{6}$$

Within the considered Bayesian framework, the choice of a given input parameter vector and its uncertainty are expressed in terms of a multivariate normal likelihood function:

$$p(\boldsymbol{v}_B|\boldsymbol{y}_B) \propto \exp(-Q_V/2), \quad Q_V = (\boldsymbol{y}_B - \boldsymbol{v}_B)^T \boldsymbol{\Sigma}_{VB}^{-1} (\boldsymbol{y}_B - \boldsymbol{v}_B).$$
 (7)

Here, v_B represents the best estimate of the input parameter vector y_B , and its uncertainty is represented by the covariance matrix Σ_{VB} . Following the same procedure as described in Hoefer & Buss (2021), the uncertainty of the input parameter vector is taken into account by drawing Monte Carlo samples δ_j^{MC} (j = 1, ..., m) from the multivariate normal pdf $N(\mathbf{0}, \Sigma_{VB})$ and subsequently replacing $y_{B,j}$ by $y_{B,j} + \delta_j^{MC}$ within the database Y before estimating the model parameters of the prior pdf $p(\mathbf{y})$. v_B is then mapped onto the best estimate $w_B = f(v_B)$ of the transformed vector z_B .

Subsequently, w_B is mapped onto the transformed response vector \mathbf{z}_A^* by applying the updating formulas of the basic MOCABA framework (see Hoefer & Buss (2021)):

$$\mathbf{z}_{A}^{*} = \mathbf{z}_{0A} + \boldsymbol{\Sigma}_{0AB} (\boldsymbol{\Sigma}_{0B} + \boldsymbol{\Sigma}_{VB})^{-1} (\mathbf{w}_{B} - \mathbf{z}_{0B})$$
(8)

$$\boldsymbol{\Sigma}_{A}^{*} = \boldsymbol{\Sigma}_{0A} - \boldsymbol{\Sigma}_{0AB} (\boldsymbol{\Sigma}_{0B} + \boldsymbol{\Sigma}_{VB})^{-1} \boldsymbol{\Sigma}_{0AB}^{T}$$
⁽⁹⁾

Here, \mathbf{z}_A^* and the corresponding covariance matrix $\mathbf{\Sigma}_A^*$ represent the model parameters of the multivariate normal posterior pdf $p(\mathbf{z}_A | \mathbf{w}_B)$ of \mathbf{z}_A :

$$p(\mathbf{z}_A|\mathbf{w}_B) = N(\mathbf{z}_A^*, \mathbf{\Sigma}_A^*) \propto \exp(-Q_A^*/2), \quad Q_A^* = (\mathbf{z}_A - \mathbf{z}_A^*)^T \mathbf{\Sigma}_A^{*-1} (\mathbf{z}_A - \mathbf{z}_A^*)$$
(10)

The posterior pdf $p(\mathbf{y}_A | \mathbf{v}_B)$ of the response vector \mathbf{y}_A is then obtained by applying the inverse transformation f^{-1} to the posterior pdf $p(\mathbf{z}_A | \mathbf{w}_B)$ of the transformed response vector \mathbf{z}_A . By expressing the information about the response vector \mathbf{y}_A conditional on the input parameter vector \mathbf{v}_B in terms of the posterior pdf $p(\mathbf{y}_A | \mathbf{v}_B)$ it is possible to quantify the quantiles of the response variables for given input parameters.

Generation and application of the meta model

Figure 2 illustrates the procedure for the generation and application of the considered meta model. The starting point is a representative database $Y_B = \{y_{B1}, ..., y_{Bm}\}$ of **observations** of the **input** parameter vector y_B . In the context of the seismic fragility analysis considered in this paper, the components of y_B represent the **seismic intensity measures** in Table 1.

To generate a database of training data for the meta model, for each observation y_{Bi} of the input parameter vector y_B , **numerical computations** are performed as described in the section "Dataset". These simulations yield a database $Y_A = \{y_{A1}, ..., y_{Am}\}$ of the response vector y_A . Combining the databases Y_A and Y_B yields the database $Y = \{y_1, ..., y_m\}$ of the combined vector $y = (y_A^T, y_B^T)^T$. The database Y is then used for **training** the meta model according to the procedure presented above. The trained meta model can

then be used to calculate in **real time** for a given input parameter vector y_B the posterior distribution of the response vector y_A .



Figure 2. Illustration of the procedure for the generation and application of the MOCABA meta model

APPLICATION TO SEISMIC FRAGILITY ANALYSIS

The described meta model procedure is now applied to a seismic fragility analysis, using a database of 180 **observations** of the 13 seismic intensity measures in Table 1 (in the following denoted as $x_1, ..., x_{13}$) and the **corresponding outcomes** of the permanent deformation h_5 of a given spacer grid (spacer grid 5).

For the training of the meta model, only those **116 samples** from the original database are used that lead to **non-zero deformations** of the considered spacer grid. The reduced database is then split in half. The **first half** (sample 1, 3, ..., 113, 115) represents the **training** data, which are used to train the meta model. The **second half** (sample 2, 4, ..., 114, 116) represents the **test** data, which are used to test the meta model. Testing the meta model means to calculate the **predictions** of the spacer grid deformation by applying successively the input parameters of sample 2, 4, ..., 114, 116 to the trained meta model and then to **compare** the obtained **posterior distributions** of the spacer grid deformation to the corresponding deformation values of sample 2, 4, ..., 114, 116. For this analysis, the uncertainties of the **input** parameters are **neglected**. The meta model is considered appropriate for quantifying probabilistically spacer grid deformations for given input parameters if the **coverage** of the test data by the credible intervals of the posterior distribution should be well covered by the test data.

Figure 3 shows the matrix of Pearson correlations between the 13 seismic intensity measures in Table 1 and the permanent spacer grid deformation h_5 . As can be seen, the average of the spectral acceleration between 1.3 and 2.4 Hz (input parameter x_7) and the average of the spectral relative velocity between 1.3 and 2.4 Hz (input parameter x_9) show the **highest correlations** (both 0.8) to the response

variable h_5 . This means that these input parameters provide a particularly high level of information regarding the prediction of h_5 . Furthermore, it can be seen that the **correlations between different input parameters** are often very high. For example, x_7 and x_9 have a correlation of 0.9. This means that there is a high degree of **redundancy** w.r.t. the information provided by the different input parameters regarding the prediction of h_5 .



Figure 3. Matrix of Pearson correlations between the 13 seismic intensity measures $x_1, ..., x_{13}$ and the permanent spacer grid deformation h_5

For the test of the meta model, we first take into account only a single input parameter, i.e. parameter x_7 representing the average of the spectral acceleration between 1.3 and 2.4 Hz. The corresponding meta model results for the posterior distribution of the permanent spacer grid deformation h_5 , obtained for the three considered distribution models, are presented in Figure 4, 5, and 6. As can be seen, the median values of the posterior distribution are fairly well covered by the test data for the lognormal model and very well covered by the log-empirical and log-Johnson models. As it should be, the 1% and 99% quantiles of the posterior distributions cover the test data for all three distribution models. However, the log-normal distribution model provides overly conservative values for the 99% quantile for high values of x_7 . In this region, the 99% quantile values appear to be overestimated by almost one order of magnitude. In comparison to the log-normal model, the log-empirical and the log-Johnson model show a significantly better performance. For these models, the quantiles of the posterior distribution follow well the slope of the test data, and the 1% and 99% quantile values are sufficiently close to the test data. The reason for the significantly better performance of the log-empirical and the log-Johnson model in comparison to the log-normal model is that these models have **more model parameters** and are, therefore, more flexible than the log-normal model. This means that these models can be well adjusted to the training data. In particular, highly non-linear dependencies between input parameters and response variables can be well described by using log-empirical and the log-Johnson distribution models.

Finally, we take into account **all** 13 input parameters for the training and testing of the meta model. Applying a log-Johnson distribution model, the corresponding results of the posterior distribution of the permanent spacer grid deformation h_5 is presented in Figure 7 as a function of input parameter x_7 . As for the single-input-parameter case, the median values of the posterior distribution are well covered by the test data. Furthermore, the test data are well covered by the intervals defined by the 1% and 99% percentiles of the posterior distribution, as it should be. Also, the 1% and 99% quantile values are sufficiently close to the test data.

Comparing the results for the single-input-parameter case in Figure 6 to the results for the 13-inputparameter case in Figure 7, shows that the outcomes for the posterior distribution of h_5 are **fairly similar** for these two cases. This could have been expected because of the **high correlation** between x_7 and h_5 and the high correlation between x_7 and other input parameters with high correlations to h_5 (see Figure 3). Because of that, input parameter x_7 **already provides a significant portion** of the information regarding the prediction of h_5 , and adding the 12 remaining input parameters does not increase the degree of information very much. In fact, the similar outcomes in Figure 6 and Figure 7 can be seen as a **consistency check** of the applied meta model procedure.



Figure 4. Posterior distributions of permanent spacer grid deformation predicted by MOCABA metamodel, using a log-normal distribution model and a single input parameter



Figure 5. Posterior distributions of permanent spacer grid deformation predicted by MOCABA metamodel, using a log-empirical distribution model and a single input parameter



Figure 6. Posterior distributions of permanent spacer grid deformation predicted by MOCABA metamodel, using a log-Johnson distribution model and a single input parameter



Figure 7. Posterior distributions of permanent spacer grid deformation predicted by MOCABA metamodel, using log-Johnson distributions and 13 input parameters

CONCLUSIONS

The presented study aims at probing the MOCABA meta-model for its **applicability** in the context of seismic safety studies.

The analysis is based on an **original** data set, consisting of seismic intensity measures (\rightarrow input data) for 180 sets of ground motion histories and corresponding spacer grid deformations (\rightarrow output data), resulting from non-linear dynamic analyses. The presented meta model uses, however, only a **censored**

dataset, consisting of those 116 samples from the original database that lead to non-zero deformations of the considered spacer grid.

For the performance tests, the **censored** dataset has been divided into a training set and a test set of **equal size**.

A general conclusion is that the MOCABA meta-model is **suitable for fragility analysis** because it can quantify the conditional probability of exceeding specific threshold values of the output parameters (seismic demand).

A more technical conclusions is that tests demonstrate that the **log-Johnson** and **log-empirical** distributions outperform the **log-normal** distribution. This has here been shown for models with a single input parameter but is generally true also for models with more input parameters.

The meta-model based on log-Johnson and log-empirical distributions leads to significantly **better** predictions of the demand distributions, especially for higher levels of ground motion. The underlying reason is that these distributions have **more model parameters** and are, therefore, **more flexible** than the log-normal model.

NOMENCLATURE

IM	Intensity measure
MOCABA	Monte-Carlo sampling and Bayes updating
PGA	Peak ground acceleration
PSA	Probabilistic safety analysis
SMA	Seismic margin assessment

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