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Division II

CURRENT STATE OF KNOWLEDGE AND DETERMINATION OF REALISTIC KE FACTORS FOR THE SIMPLIFIED ELASTIC - PLASTIC FATIGUE ANALYSIS

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INTRODUCTION

The aim of the presentation of the research project, Trieglaff et al. (2020), is to address the current state of knowledge on the use of ke factors in the context of simplified elastic - plastic fatigue analyses. If the simplified elastic-plastic fatigue analysis is used, the influence of plastic deformations must be taken into account in the case of over-elastic loading by using the strain correction factor ke. The ke factor is determined for certain material groups by simple calculation formulas depending on the stress level.

This procedure was originally anchored in the American nuclear code ASME III and is also implemented in the German nuclear code KTA. The essential basic ideas for the derivation of the formulas for determining the ke factor are presented in order to show the conceptual limitations. These are justified by the simple mechanical model approaches and assumptions for application and the maximum value of the ke factor. This approach usually leads to very conservative calculation results. However, non-conservative results can also occur in the area of low plastic strains and for components with strong notches.

Based on an extensive literature study, fundamental suggestions for improvement from the current literature and selected international codes are presented. These are essentially characterised by the separation of the consideration of mechanical and thermal expansion factors, the definition of a weighted superimposition rule, the inclusion of the excessive expansion on component notches in the case of thermal stress and the retention of the original formation rule in the case of mechanical stress.

In order to reduce conservatism in the assessment procedure, calculations based on elastic-plastic material behavior with the aim of a realistic determination of the strain range are used more often in the last years. The results of elastic-plastic FE analyses taken from the literature and in addition the results of the finite element calculations with different geometry variants are presented and compared with calculation procedures of different nuclear codes.

Finally, we give our assessment of the applicability of the different rules and the potential for determining realistic strain ranges. We develop a suggestion for further improvement of the nuclear codes for the calculation of fatigue analyses.

EVALUATION OF LITERATURE

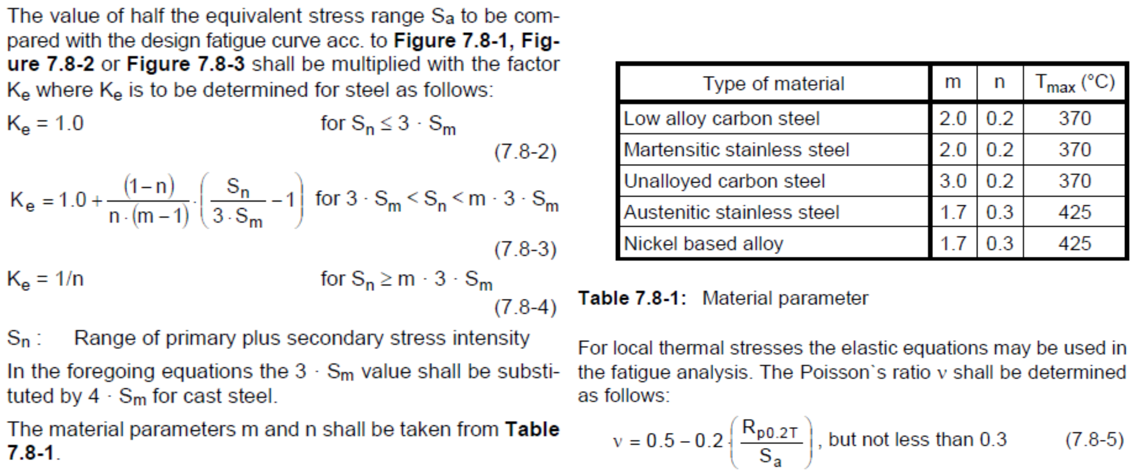
According to the nuclear protection objectives, the removal of residual heat and thus the preservation of the coolant inventory within the pressure-retaining enclosure as well as the confinement of radioactive substances have to be ensured. For this purpose, it has to be verified on the basis of the nuclear rules and regulations, here KTA 3201.2 (November 2017) and KTA 3211.2 (November 2013), that the pressure-retaining walls of components in nuclear power plants withstand all specified mechanical and thermal loads during design and all loads actually occurring during operation (load level and frequency).

The verification against cyclic loads, in particular mechanical and thermal transients, is carried out on the basis of a fatigue analysis. The evaluation basis for the fatigue analysis are fatigue curves based on mainly strain-controlled tests of small samples in air atmosphere.

Different methods are used for fatigue analysis.

The **simplified elastic-plastic fatigue analysis** may be used if the equivalent stress range of all primary and secondary stresses (i.e. mechanical and thermal stresses) exceeds the limit $3S_m$ (S_m = stress equivalent value from KTA 3201.2) for components made of steel and $4S_m$ for components made of cast steel, but these limits are met by the equivalent stress range of the primary and secondary stresses due to mechanical loads. If the simplified elastic-plastic fatigue analysis is applied, the influence of plastic deformations must be taken into account in the case of overelastic loading by using the strain increase factor ke. The ke factor is determined for certain material groups by simple calculation formulas depending on the load condition. However, experimentally or mathematically proven values or values taken from the literature can also be used.

The procedure for the application of the simplified elastic-plastic fatigue analysis is specified in KTA 3201.2 (November 2017) in Chapter 7.8.4 (Figure 1):



The value of half the equivalent stress range S_a to be compared with the design fatigue curve acc. to **Figure 7.8-1, Figure 7.8-2** or **Figure 7.8-3** shall be multiplied with the factor K_e where K_e is to be determined for steel as follows:

$$K_e = 1.0 \qquad \text{for } S_n \le 3 \cdot S_m \qquad (7.8\text{-}2)$$

$$K_e = 1.0 + \frac{(1-n)}{n \cdot (m-1)} \cdot \left(\frac{S_n}{3 \cdot S_m} - 1 \right) \qquad \text{for } 3 \cdot S_m < S_n < m \cdot 3 \cdot S_m \qquad (7.8\text{-}3)$$

$$K_e = 1/n \qquad \text{for } S_n \ge m \cdot 3 \cdot S_m \qquad (7.8\text{-}4)$$

S_n : Range of primary plus secondary stress intensity

In the foregoing equations the $3 \cdot S_m$ value shall be substituted by $4 \cdot S_m$ for cast steel.

The material parameters m and n shall be taken from **Table 7.8-1**.

Type of material	m	n	T_{max} (°C)
Low alloy carbon steel	2.0	0.2	370
Martensitic stainless steel	2.0	0.2	370
Unalloyed carbon steel	3.0	0.2	370
Austenitic stainless steel	1.7	0.3	425
Nickel based alloy	1.7	0.3	425

Table 7.8-1: Material parameter

For local thermal stresses the elastic equations may be used in the fatigue analysis. The Poisson`s ratio ν shall be determined as follows:

$$\nu = 0.5 - 0.2 \left(\frac{R_{p0.2T}}{S_a} \right), \text{ but not less than } 0.3 \qquad (7.8\text{-}5)$$

Figure 1: KTA 3201.2 – Chapter 7.8.4

The **general elastic-plastic fatigue analysis** is based on elastic-plastic material behaviour in deviation from the above methods, whereby it must additionally be shown that no failure occurs as a result of progressive deformation.

The method for determining the ke factors according to KTA 3201.2 (November 2017) and KTA 3211.2 (November 2013) is based on the specifications of the American nuclear rules and regulations ASME BPVC.III.1.NB (2017). In the report EPRI Technical Report (2018), the historical development of this procedure of the ASME code is presented.

In EPRI Technical Report (2018) it is stated that the technical basis for the simplified elastic-plastic analysis implemented in ASME Code Section III is based on a method originally developed by Langer (1971).

From two calculation models he derived the basis for the formulas for determining the strain increase factor ke. These are

a) a tapering flat bar under tensile load and
b) a cantilever beam.

According to the EPRI Technical Report (2018), Tagart proposed a modification of Langer's proposal in Tagart (1968), according to which the generic values determined by Langer should not necessarily apply to all materials. He proposed different values based on the ANSI / USAS B31.7 (1969) rules. The values were differentiated between stainless steel, low alloy steel and carbon steel. This resulted in the expression for ke, which was also adopted for KTA Safety Standards 3201.2 (November 2017) and 3211.2 (November 2013).

As shown, the method for determining the ke factor is based on simple mechanical models and simplified specifications. Weaknesses soon became apparent and discussions ensued that led to proposals and modified specifications in regulations. These alternative methods aim both at reducing conservatism and at building up conservatism in the case of effects that are not taken into account.

Based on the comparison of the results of the simplified elastic-plastic analysis with those of a detailed elastic-plastic analysis, the following points should be mentioned with regard to the reduction of conservativities:

- Separation of the consideration of mechanical and thermal strain increase factors
- Reduction of the currently used mechanical strain-increase factors
- Definition of thermal strain-increase factors
- Definition of a suitable superposition rule of mechanical and thermal strain-increase factors

With regard to building up conservatism in the area of low stress ranges and in the area of component notches, the following points are discussed:

- Validity of the limit ($S_n/3S_m = 1$) up to which the ke factor is fixed at 1.
- Level of the ke factor in the immediate area of this limit ($S_n/3S_m = 1$)
- Addition of an application rule for component notches
- Formation rule of the ke factors: Use of the secondary stress range S_n (linearised stress curve) or the peak stress range S_p (non-linear stress curve = total stresses).

Several publications have proposed improvements to the rules of the ASME Code for simplified elastic-plastic analyses in order to avoid the weak points of the simplified definition of the ke factor. The suggestions for improvement refer to a reduction of the conservatism in the determination of the ke factors, especially in the case of thermal transients, and the increase of the conservatism in the cases where a notch effect has to be considered. The suggestions for improvement are described in detail in Trieglaff et al. (2020) and are only mentioned here:

- Welding Research Council Bulletin (WRC) 361, Grandemange et al. (1991).
- EPRI (1998) methodology for simplified elastic-plastic analysis
- ASME Code Case N-779 (2009)
- New proposal according to Reinhard and Ranganath (2018)
- Calculation on the basis of the simplified yield zone theory (Hübel (2015), Hübel (2016) and Hübel et al. (2014))

In addition, the methods for determining the correction factors for the simplified elastic-plastic fatigue analysis are presented in Trieglaff et al. (2020) for further international nuclear rules in the form of the French code RCC-M (Grandemange et al. (1991)) and the Japanese code JSME (Asada et al. (2010)). Comparatively, the methods of the German regulations for unfired pressure vessels AD 2000-Merkblatt S2 (December 2012) and the corresponding European regulations EN 13445-3 Chapter 18 (December 2018) are also presented.

Currently, the verification is also increasingly carried out on the basis of elastic-plastic material behaviour with the aim of a realistic determination of the strain range in order to reduce conservatism in the verification. This type of verification is increasingly used to verify that the "attention thresholds" defined in the currently valid versions of KTA 3201.2 (November 2017) and KTA 3211.2 (November 2013) are not exceeded in order to take into account the influence of the medium with regard to fatigue damage.

In many codes of practice, the possibility of carrying out a verification on the basis of elastic-plastic calculations is opened up, but a precise description of the procedure is rarely given. An exception is ASME VIII.2 (2017) as a set of rules for conventional pressure equipment, where the determination of the ke factor on the basis of elastic-plastic material behaviour is described.

In the research project, results of elastic-plastic FE analyses were collected from the literature and used for the verification or discussion of the suggestions for improvement and the presented code methods for determining ke.

These calculations include partly very simple academic, but also quite realistic examples. The examples also differ in the material law used (with and without hardening). In some cases, only mechanical loads or only thermal loads (mostly temperature jumps) were analysed. In some cases, also a combination of both was considered. Notched and unnotched components were also analysed.

The corresponding data points were compared with the KTA (ASME) code curves for austenite and for Poisson's correction when analysing thermal transients alone and the curve from the French code RCC-M for thermal cyclic stresses. Except for the range of low strains, the curve for austenite covers the calculation results very conservatively. On the other hand, a Poisson's correction alone is not considered to cover the analysed examples. The control curve from the RCC-M presents itself as an upper limit curve and also covers the results in the range of low strains.

A corresponding comparison was also made with regard to the calculation results for analyses based on stress from purely mechanical as well as in combination with thermal transients. A comparison with the KTA (ASME) curve for austenite shows that it covers almost all results including notch effect.

EXAMPLE CALCULATIONS

Within the project, ke factors were derived and compared for three different components (reducer, nozzle, pipe bend), each with two geometry variants, on the basis of elastic FE calculations according to the standard procedure of KTA/ASME, the new proposal of 2018, the French code of practice RCC-M and the European code of practice EN 13445-3. Furthermore, the determined ke factors of the simplified elastic-plastic fatigue analysis were compared with the results of the performed elastic-plastic FE calculations for selected geometry variants.

The results of the calculation examples confirm the conservativeness of the standard method ($ke_{KTA/ASME}$) compared to other methods and the results of an elastic-plastic calculation. The results of the "New Proposal" of 2018 are less conservative compared to the standard method and partly show a good tendential agreement with the results of the elastic-plastic calculation. For the case of dominating thermal stresses, the results of the RCC-M method can be evaluated as close to reality, as they cover the results of the elastic-plastic calculations without large conservatism. This also applies to linearised stress ranges in the range of low strains. The results for the ke factor on the basis of EN 13445-3 show the lowest values and are partly below the results of the elastic-plastic calculation. However, it should be pointed out that this set of rules is based on different fatigue curves compared to the nuclear rules used here and therefore only a comparison of the permissible number of load cycles allows a conclusive evaluation (Figure 2 & 3).

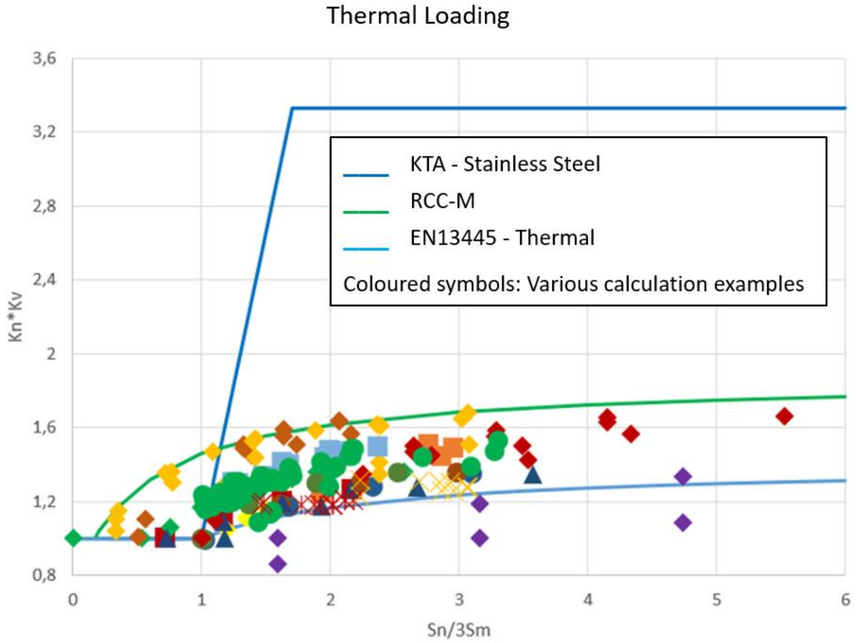


Figure 2: Thermal loading - Trieglaff et al. (2020)

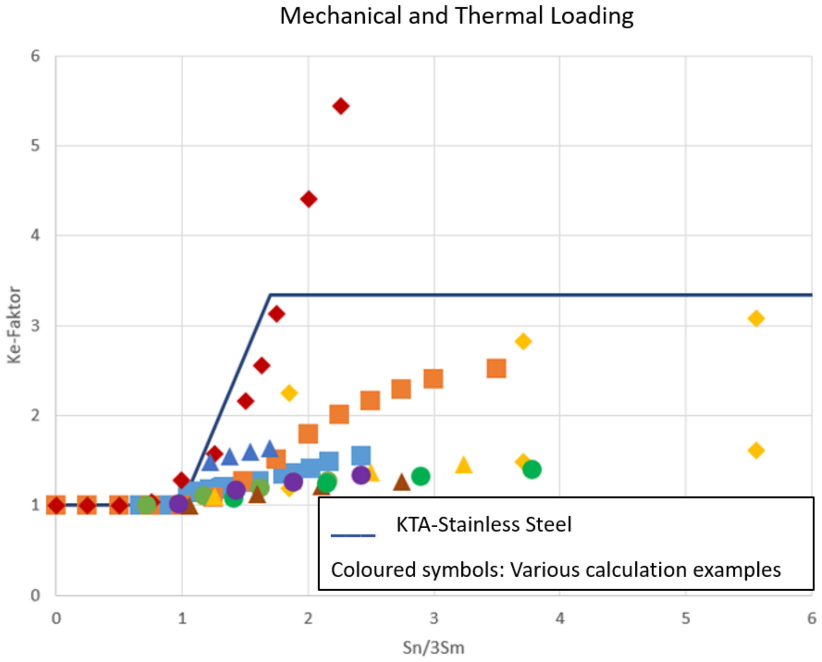


Figure 3: Mechanical and thermal loading - Trieglaff et al. (2020)

EVALUATION WITH REGARD TO THE KTA SAFETY STANDARDS

Conservativeness Of The Procedure

Based on the evaluation of various publications, codes of practice and proposed codes of practice as well as our own calculations, we can derive the following statements regarding the current specifications in the KTA safety standards for the determination of the strain increase factor ke:

The calculation results from the literature as well as our own calculations confirm the conservativeness of the current specifications in the KTA safety standards for determining the strain increase factor ke. This applies under the conditions that the linearised total stress range S_n clearly exceeds the value of $3S_m$, the condition for limiting the mechanical stress range of $3S_m$ is fulfilled and the thermal bending stresses are dominant. From our experience from the nuclear supervision procedure, these conditions are usually met for the fatigue-relevant load cases (thermal shock and thermal stratification).

If in elastic-plastic calculations the material behaviour is not defined as ideal elastic-plastic, but is described realistically on the basis of cyclic stress-strain curves, this also implies plastic strains below the yield point. This applies in particular to components made of austenitic material. Experiences from the nuclear supervisory procedure show that the results for the fatigue load transients of components of the primary and secondary circuit were hardly determined in this order of magnitude of the linearised equivalent stress range $S_n \approx 3S_m$. The need for an elastic-plastic calculation to comply with the permissible fatigue life usage factor D was mostly justified by the requirement to apply maximum values for ke factors (3.3 for austenite or 5.0 for ferrite) on the basis of elastic calculations. Here, a targeted re-evaluation of existing calculations for fatigue-bearing components in a follow-up project to this research project could support this argumentation.

Consideration Of Notches

With regard to the evaluation of notches in pressure-bearing components, this discussion is conducted separately for mechanical and thermal loads in the evaluated documents.

For mechanical loads, the standard formula for ke, as implemented in the German and American codes, is considered to be sufficiently conservative, so that common notches are covered. The limitation of the equivalent stress range from primary and secondary membrane stresses and bending stresses without thermal bending stresses over the wall thickness with $3S_m$ for steels and with $4S_m$ for cast steel in the KTA is a necessary condition. In addition, notches in highly stressed component areas should be nearly excluded in German nuclear facilities, whose safety-critical systems and components are designed according to the principle of structural basic safety.

In the following, the determined stress increases are described on the basis of linear-elastic FE calculations in the area of notches with so-called form factors. The fatigue-damaging effect of notches can be assigned to the long-term strength range in its full amount. Investigations in the fatigue strength range and especially in the short-term strength range show a clear reduction of this fatigue-damaging effect. This is taken into account by a load cycle-dependent correction on the basis of an effective notch factor in EN 13445-3. However, the available database for pressure-bearing components in publications is too small to derive a generally valid correction of the notch effect in the short-term strength range. This could be one reason why this correction has not yet found its way into the other codes considered here. Additional research activities would be desirable here.

As the results of elastic-plastic calculations shown in Figures 2 (Trieglaff et al. (2020)), which also take usual notches into account, prove, the resulting increase in strain is largely covered by the standard ke factor of the KTA.

Peak Stress Range Sp As Basis Of The ke Factor

Another point of discussion is that the fatigue analysis in nuclear codes for the determination of permissible load cycles or degrees of utilisation are based on peak stress ranges S_p, but the elastic-plastic correction ke Factor is based on linearised stress ranges S_n. This has a negative effect on the calculation of the ke Factor. This has the consequence, for example, in the calculation of thermal shocks, that the highest peak stresses occur at the beginning of the transient and a quasi-stationary temperature distribution over the wall only occurs in the course of the transient and thus the highest linearised thermal stresses. In this case, the times of the calculated maximum peak stresses differ from those of the maximum linearised stresses. Here, the user of the corresponding regulations is not given a clear guideline for action. A conservative approach is to correct the maximum peak stress range S_p with the maximum ke factor on the basis of the linearised stress range S_n, even if these stress ranges do not occur at the same time. This simplifies the procedure and reduces the calculation effort.

Another way to eliminate this problem is to determine ke factors on the basis of peak stress range transients S_p, as implemented, for example, in the Japanese nuclear rules and regulations. However, this requires a fundamentally different procedure. The curve representing the ke factor is based on a large number of elastic-plastic calculations of typical nuclear components and represents a limit curve of the results.

With regard to a harmonisation of the application, at least a textual supplement in the form of a clear guideline for action would make sense for current safety standards if the times of the calculated maximum peak stresses deviate from those of the maximum linearised stresses.

Superposition Of Thermal And Mechanical Stresses

The separate consideration of thermal and mechanical stresses in many codes and improvement proposals raises the question of a suitable superposition rule. In the considered codes and proposals, these specifications range from the sole application of the ke factor for mechanical loads, in the case of simultaneous occurrence of thermal and mechanical loads over the additive superposition of the components from thermal and mechanical loads with their own strain correction factors and up to a weighted superposition of the components from thermal and mechanical loads with their own strain correction factors.

However, the associated reduction of conservatism increases the complexity of the application of the calculation rules, since the linearised stress ranges have to be formed here with regard to their load cause. This procedure increases the calculation effort for the user, but also the error-proneness in the interpretation of the formation rules for the strain correction.

This may also be one of the reasons why concepts for a separate evaluation of thermal and mechanical stresses were discussed during the last revision of KTA 3201.2 and KTA 3211.2, but these were not included in the corresponding draft safety standards. This question does not arise in the current procedure of simplified elastic-plastic fatigue analysis.

Elastic-Plastic Calculations

In general, formation rules for strain correction based on the results of elastic-plastic calculations (as in the French and Japanese nuclear codes) have the best agreement with the results of elastic-plastic calculations.

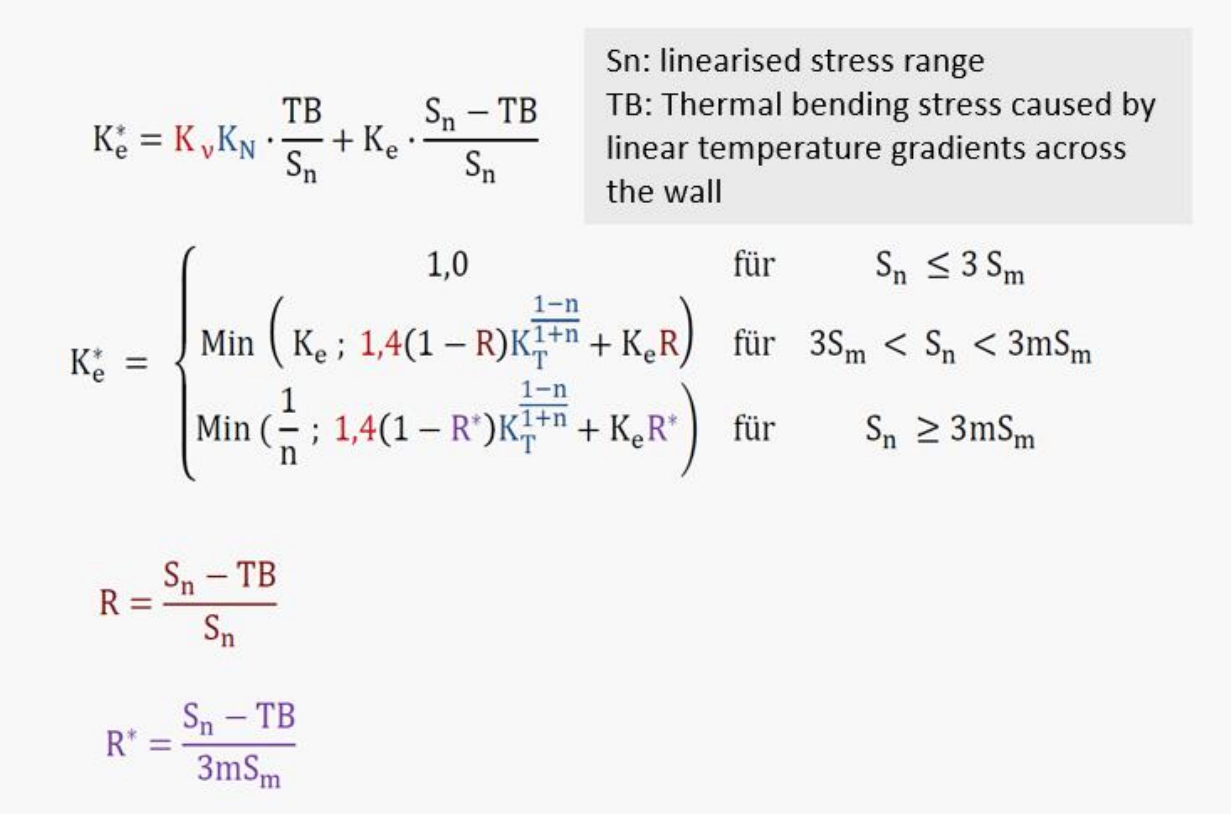
It should be noted that the procedure for elastic-plastic analyses is hardly regulated in the considered codes, with the exception of the conventional ASME VIII-2 code. Here, specifications for the user regarding the formulation of the material law, the flow condition and the strain hardening parameters would be desirable.

In the short-term strength range, failure due to progressive plastic deformation (ratcheting) must be considered in addition to fatigue failure. In the case of the concrete regulation of the performance of fatigue analyses on the basis of elastic-plastic calculations, this should be in accordance with the verification against thermal ratcheting.

Furthermore, the database of material tests on which the fatigue curves in the short-term strength range are based is generally not very comprehensive. The definition of the ke factor should generally be in accordance with a corresponding database. Meaningful tests of small specimens up to test specimens that are re-presentative of real components could serve to further increase knowledge.

EVALUATION OF THE PRESENTED METHODS

Many international research projects have been performed in connection with the lifetime extension of existing nuclear facilities under consideration of the fatigue-damaging influence of the medium. The aim is to reduce conservatism based on a re-evaluation of fatigue analyses already carried out in the past. Therefore, no general change of approach can be observed in the presented proposals for improvement. However, the results of the new proposal according to Reinhard and Ranganath (2018) (figure 4 & 5) show that they are less conservative compared to the standard procedure of the KTA/ASME and partly show a good agreement with the results of the elastic-plastic calculation. Therefore, this method seems to be suitable as an improvement compared to the standard method, even if it does not eliminate all points of criticism.



$$K_e^* = K_\nu K_N \cdot \frac{TB}{S_n} + K_e \cdot \frac{S_n - TB}{S_n}$$

Sn: linearised stress range
TB: Thermal bending stress caused by linear temperature gradients across the wall

$$K_e^* = \begin{cases} 1{,}0 & \text{für} \quad S_n \leq 3\,S_m \\ \mathrm{Min}\left(K_e\,;\ 1{,}4(1-R)K_T^{\frac{1-n}{1+n}} + K_e R\right) & \text{für} \quad 3S_m < S_n < 3mS_m \\ \mathrm{Min}\left(\frac{1}{n}\,;\ 1{,}4(1-R^*)K_T^{\frac{1-n}{1+n}} + K_e R^*\right) & \text{für} \quad S_n \geq 3mS_m \end{cases}$$

$$R = \frac{S_n - TB}{S_n}$$

$$R^* = \frac{S_n - TB}{3mS_m}$$

Figure 4: New approach by Reinhard and Ranganath (2018)

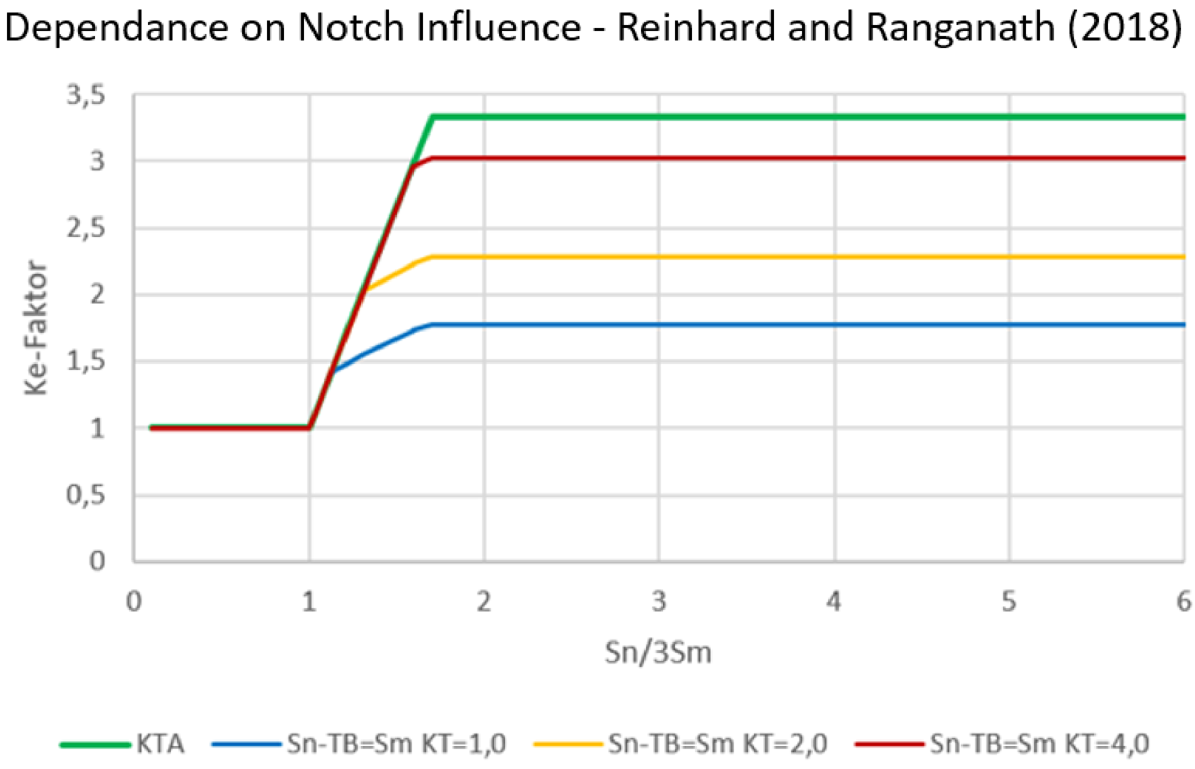


Figure 5: Exemplary results in comparison to KTA (Trieglaff et al. (2020))

An alternative procedure is the calculation based on the simplified yield zone theory. In the appendix of KTA 3201.2 (November 2017), reference is made to the application of the simplified yield zone theory as a possible alternative for the ratcheting verification (Hübel (2015), Hübel (2016) and Hübel et al. (2014).

With regard to the code procedures, the procedure in the French code for the evaluation of thermal stresses shows good agreement with the results of the elastic-plastic calculations.

In order to determine realistic results in the simplified elastic-plastic fatigue analysis, the procedure in the Japanese Code Case NC-CC-005 (Asada et al. 2010) is useful. The calculation of the correction factors using the peak stress range represents a simplification in the application. The basis for this is the generation of a covering curve based on a large number of elastic-plastic calculations.

CONCLUSION

With the investigations carried out within the presented research project, we could demonstrate that existing regulations are generally conservative with regard to the ke factors in the context of fatigue analyses approach. However, there are several approaches to improve the existing regulations. Close to the existing rules, the new proposal according to Reinhard and Ranganath (2018) appears to be an option. Through this, conservatism is reduced and notches are explicitly taken into account. The adoption of the regulation from the French nuclear regulations especially for thermal loading shows a good agreement with elastic-plastic calculations. The procedure of the simplified yield zone theory according to Prof. Hübel can also be seen as an additional option. The procedure in the Japanese Code Case NC-CC-005 regarding the performance and evaluation of a large number of elastic-plastic calculations of representative nuclear components could also be transferred to other nuclear facilities.

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