Transactions, **SMiRT-26**
Berlin/Potsdam, Germany, July 10-15, 2022
Division 04

ENERGY-BALANCED INTENSITY OF EARTHQUAKE GROUND MOTIONS

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ABSTRACT

The Arias intensity is extensively applied as a criterion for correlating strong ground motion to earthquake destructiveness or structural damage. An energy-balanced index is derived and expressed in terms of the ground accelerogram, structural damping ratio and frequency. An investigation is performed for the relationships of earthquake input energy, structural kinetic energy, dissipated energy due to damping, and elastic strain energy based on the input of the 1940 El Centro strong earthquake motion. It is shown that the earthquake input energy is balanced by the structural kinetic energy plus strain energy instead of the dissipated energy during the strong earthquake excitation, but after the earthquake event the input energy is balanced by the dissipated energy due to damping for an elastic system. The derived intensity can be applied to determine the strong motion duration and to design seismic sensors for the seismic instrumentation and qualification of safety-related structures.

INTRODUCTION

In seismic design and qualification of safety-related structures, systems and components, it is crucial to determine the parameters of design-basis earthquake. In this study, energy-balanced earthquake intensity with energy-balanced seismic response is studied based on the Arias intensity (Arias 1970). A brief review of Arias intensity applications is performed first. An assessment of the derivation of Arias intensity is carried out to indicate that the expression of Arias intensity leads to unbalanced dissipation energy. Then, an expression of intensity with balanced earthquake energy is derived based on a Single Degree of Freedom (SDOF) structural system. Based on the derived earthquake intensity, the effect of upper bound frequency on the prediction of earthquake intensity is investigated. In addition, the energy-based intensity is applied to show the energy response of seismometers. Finally, the derived intensity expression is applied on the 1940 El Centro acceleration time histories (Liu and Lu 2010) to study its strong earthquake duration.

A Brief Review of Earthquake Damage Intensity

Structural damage of structures subject to earthquake loading is related to seismic level or earthquake intensity. There are many parameters that can be used as earthquake intensity for structural damage measures, including peak ground acceleration (PGA), root mean square (RMS) acceleration, Housner intensity, peak ground velocity (PGV), peak ground displacement (PGD), spectral response acceleration (SRA), and Arias intensity. It is logically recognized that the energy-based intensity is much more meaningful because the amplitude, duration, and frequency contents of the earthquake strong motion are taken into account. In light of the basic assumptions for the SDOF system subjected to ground strong motion, the energy-based intensity is derived in this section. It is assumed that the ground input energy is balanced with the damping energy that is used to characterize the structural damage. Thus, the earthquake input energy rather than the damping energy itself is applied for the derivation of the earthquake intensity, which is expressed in terms of structural frequency, damping ratio, and the time history of earthquake ground acceleration.

Basic Assumptions

It is difficult to derive a general earthquake intensity accounting for structural damage associated with stiffness degradation because structural failure due to plasticity should be determined on a case-by-case basis. Thus, a general intensity characterizing structural damage or earthquake destructiveness should exclude the influence of the structural plasticity (Arias 1970). The structure under consideration for the measure of earthquake strength must be simple and behave elastically to avoid conducting complicated dynamic analysis associated with geometrical and material nonlinearity. Nevertheless, a minimum of structural properties should be considered to obtain the measure of earthquake intensity. As a result, the following assumptions are made for the derivation:

i. A structure is idealized as a SDOF system with circular frequency ranging from 0 to ϑ of interest;
ii. Structural damage degree is proportional to the dissipated energy per unit weight of the structure;
iii. The dissipated energy is characterized by the viscous damping energy;
iv. Each structure affected by the earthquake motion behaves elastically without plastic energy loss; and
v. Earthquake intensity is defined as the energy dissipation accumulated in the domain of structural frequency.

Following these assumptions, a SDOF structural system can be considered in deriving the energy-based earthquake intensity.

Concern for Arias Intensity

For a SDOF system consisting of a lumped mass m, viscous damping coefficient c, and linear stiffness k, which is enforced by earthquake ground motion y_0, the structural motion can be expressed in terms of the relative motion u as (Liu and Lu 2010):

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{y}_0 \tag{1}$$

which is used to derive the intensity. The balanced-energy equation can be defined as the work done by the force along the displacement u during the earthquake excitation t_d by integrating Equation 1 as:

$$\int_0^{t_d} \ddot{u}du + 2\beta\omega\int_0^{t_d} \dot{u}du + \omega^2\int_0^{t_d} udu = -\int_0^{t_d} \ddot{y}_0 du \text{ or } E_k + E_d + E_s = E_i \tag{2}$$

where β, ω, and y_0 are the viscous damping ratio, circular frequency, and base displacement for the SDOF system. Terms E_k, E_d, E_s, and E_i in Equation 2 represent kinetic, dissipated, strain, and input energies, respectively. From the assumptions above, after the earthquake event, the kinetic energy E_k, and strain energy E_s vanish or equal to zero; and the dissipated energy of the structure is equal to the earthquake input energy:

$$2\beta\omega\int_0^{t_d} \dot{u}du = E_d = E_i = -\int_0^{t_d} \ddot{y}_0 du \tag{3}$$

Upon using the right integral of earthquake input energy in Equation 3 and the solution of Equation 1, the following intensity expression has been derived (Arias 1970):

$$I_a = \frac{\arccos(\beta)}{\sqrt{1-\beta^2}}\int_0^{t_d} \ddot{y}_0^2 dt = C_a(\beta)\int_0^{t_d} \ddot{y}_0^2 dt \tag{4}$$

where the gravity constant g is omitted from the original Arias' derivation (1970). Term C_a is a function of damping ratio β. When the damping value β tends to zero, Equation 4 is standardized as:

$$I_a(0) = \frac{\pi}{2}\int_0^{t_d} \ddot{y}_0^2 dt \tag{5}$$

Sensitivity analysis shows that the variation of damping value β in the interesting range of earthquake engineering does not change the I_a value significantly; thus, using Equation 5 would not depart too much from I_a values associated with moderate damping ratios (Arias 1970).

A concern is raised for the correctness of derivation for Equation 4 that might violate the assumptions iii and v above. When $\beta = 0$, the left integral in Equation 3 should be zero; i.e., the dissipated energy $E_d = 0$, and the earthquake input energy E_i cannot be balanced by the zero dissipated energy. Then, the earthquake intensity cannot be defined as the energy dissipation accumulated in the structural frequency domain.

The issue in the original derivation (Arias 1970) might be due to the determination of the lower and upper bounds/limits for the triple integral of the earthquake input energy E_i during changing the order of integration. More details can be found on Page 481 in Appendix of the reference (Arias 1970). A new derivation is presented in the following section.

DERIVATION OF ENERGY-BALANCED INTENSITY

Energy Expression for Earthquake Intensity

From Assumption v above, the earthquake intensity should be expressed based on the dissipated energy E_d accumulated in the structural frequency domain. If the displacement u in Equation 1 is found by solving the dynamic Equation 1, then the dissipated energy due to viscous damping is defined as:

$$E_d = \int_{u_{\min}}^{u_{\max}} c\dot{u}du = \int_0^{t_d} c\dot{u}^2 d\tau \tag{6}$$

where $u_{\min}$ and $u_{\max}$ are respectively the low and upper limits of displacement within the time duration t_d being considered. The middle term of Equation 6 is the accumulated work done by the damping force along the corresponding displacement. It is not easy to evaluate this energy because the velocity $\dot{u}$ is a multiple-value function of displacement u. Therefore, the integral with respect to u is transformed into that with respect to time t as the rightmost term in Equation 6, from which the energy-based intensity I_d can be expressed by the cumulative energy in the frequency domain as:

$$I_d = \int_0^{\vartheta} \frac{E_d}{m} d\omega \tag{7}$$

where ω is the circular frequency of the system and is given by:

$$\omega = \sqrt{k / m} \tag{8}$$

The upper frequency limit ϑ of the integral in Equation 7 is the maximum circular frequency to be considered, whose value was assumed infinity in the derivation (Arias 1970). To avoid directly using E_d, the input energy E_i Equation 3 is modified as:

$$E_d = E_i = -\int_0^{t_d} \ddot{y}_0 du = -\int_0^{t_d} \ddot{y}_0 \dot{u} dt \tag{9}$$

which is applied in the following derivation.

Response of SDOF Structural System

The displacement of a SDOF system can be expressed as (Chopra 2001):

$$u = -\frac{1}{\omega_d} \int_0^t \ddot{y}_0(\tau) e^{-\beta\omega(t-\tau)} \sin[\omega_d (t-\tau)] d\tau \tag{10}$$

where y_0 is a continuous function with regard to time variable t. Term τ is a time variable similar to t. Term ω_d is the damped circular frequency with expression

$$\omega_d = \omega\sqrt{1-\beta^2} \tag{11}$$

Derivation of Earthquake Intensity

In deriving the earthquake intensity, it is required to find the displacement increment du or the corresponding velocity. To this end, differentiating Equation 10 with respect to time t, the velocity of the SDOF system is given after rearrangement by

$$\dot{u} = -\int_0^t \ddot{y}_0 e^{-\beta\omega(t-\tau)} \left\{ \cos[\omega_d (t-\tau)] - \frac{\beta\omega}{\omega_d} \sin[\omega_d (t-\tau)] \right\} d\tau \tag{12}$$

which is the ratio of displacement increment du to increment time dt. Note that the low limit of time was extended from 0 to -∞ in the Arias derivation (Arias, 1970) but it is unnecessary to do so. According to the assumptions made for the earthquake intensity, the structural frequency ω is not expected to appear in the expression. To this end, an integration over the frequency range from 0 to ϑ is conducted to eliminate the frequency ω. Therefore, the earthquake intensity per unit mass over the structural frequency range is defined by:

$$I_d = \int_0^{\vartheta} \frac{E_d}{m} d\omega = \int_0^{\vartheta} \left(2\beta\omega \int_0^{t_d} \dot{u}^2 dt \right) d\omega = -\int_0^{\vartheta} \left(\int_0^{t_d} \ddot{y}_0 \dot{u} dt \right) d\omega \tag{13}$$

Substituting for the velocity in Equation 12 into Equation 13 yields:

$$I_d = \int_0^{\vartheta} \left(\int_0^{t_d} \ddot{y}_0(t) \int_0^t \ddot{y}_0(\tau) e^{-\beta\omega(t-\tau)} \left\{ \cos[\omega_d (t-\tau)] - \frac{\beta\omega}{\omega_d} \sin[\omega_d (t-\tau)] \right\} d\tau dt \right) d\omega \tag{14}$$

In order to evaluate this complicated integral, the following two parameters are introduced:

$$a = -\beta(t-\tau);\ b = \sqrt{1-\beta^2}(t-\tau) \quad \text{15a, b)}$$

and then changing the order/sequence of the integration in Equation 14 can yield

$$I_d = \int_0^{t_d} \ddot{y}_0(t) \int_0^{t_d} \ddot{y}_0(\tau) \int_0^{\vartheta} e^{a\omega}[\cos(b\omega) + \frac{a}{b}\sin(b\omega)]d\omega d\tau dt = \int_0^{t_d} \ddot{y}_0(t) \int_0^{t} \frac{\ddot{y}_0(\tau)}{b}(bA + aB)d\tau dt \quad (16)$$

where terms A and B are two intermediate parameters that are two integrals as defined in the following. Once these two integrals are determined, the intensity in Equation 16 can be further simplified. To that end, the formulas provided on page A59 of Reference (Grossman, 1981) are applied to get the A and B expressions as below

$$A = \int_0^{\vartheta} e^{a\omega}\cos(b\omega)d\omega = \frac{e^{a\vartheta}[a\cos(b\vartheta) + b\sin(b\vartheta) - a}{a^2 + b^2} \quad (17)$$

$$B = \int_0^{\vartheta} e^{a\omega}\sin(b\omega)d\omega = \frac{e^{a\vartheta}[a\sin(b\vartheta) - b\cos(b\vartheta) + b}{a^2 + b^2} \quad (18)$$

According to the expressions in Equations 17 and 18, the term $bA + aB$ in the integrant of Equation 16) can be expressed after rearranging and simplifying as,

$$bA + aB = e^{a\vartheta}\sin(b\vartheta) \quad (19)$$

which is quite a simple expression for the complicated integral because some terms are cancelled out. Substituting Equation 19 into Equation 16, the integral with respect to variable τ can be evaluated using integration by parts (Grossman 1981) as follows:

$$\begin{aligned} \int_0^t \frac{\ddot{y}_0}{b}(bA + aB)d\tau &= \int_0^t \frac{\ddot{y}_0}{b} e^{a\vartheta}\sin(b\vartheta)d\tau = \int_0^t \frac{\ddot{y}_0}{\beta\vartheta\sqrt{1-\beta^2}(t-\tau)}\sin[\sqrt{1-\beta^2}(t-\tau)\vartheta]de^{-\beta\vartheta(t-\tau)} \\ &= \frac{\ddot{y}_0 e^{-\beta\vartheta(t-\tau)}}{\beta\vartheta\sqrt{1-\beta^2}(t-\tau)}\sin[\sqrt{1-\beta^2}(t-\tau)\vartheta]\Big|_0^t - C = \frac{\ddot{y}_0}{\beta} - C \end{aligned} \quad (20)$$

Note that when using the upper limit ($\tau \to t$) condition, the limiting value $\sin\varepsilon/\varepsilon = 1$ is applied, because of the small value of $\varepsilon = \sqrt{1-\beta^2}(t-\tau)\vartheta$ for $\tau = t$; when using the low limit, $\ddot{y}_0(0) = 0$ is applied, which means that the acceleration should be zero at the start time. Term C in Equation 20 is also a complicated expression and can be evaluated by using the mean value theorem of integral described on page 319 of the Reference (Grossman 1981) as:

$$C = \int_0^t e^{-\beta\vartheta(t-\tau)}F(\tau)d\tau = F(t,\xi)\frac{e^{-\beta\vartheta t} - 1}{\beta\vartheta} \quad (21)$$

$$F(t,\xi)=\left[\frac{\ddot{y}_0}{t-\xi t}+\frac{\dot{y}_0}{(t-\xi t)^2}\right]\frac{\sin[\sqrt{1-\beta^2}(t-\xi t)\vartheta]}{\beta\vartheta\sqrt{1-\beta^2}}-\frac{\dot{y}_0}{\beta(t-t\xi)}\cos[\sqrt{1-\beta^2}(t-\xi t)\vartheta] \quad (22)$$

in which ξ is a value within the range of 0 to 1. Now substituting Equation 20 back into Equation 16, the energy-based intensity is finally expressed in a simplified form as:

$$I_d=\int_0^{t_d}\ddot{y}_0\left[\frac{\ddot{y}_0}{\beta}-C\right]dt \quad (23)$$

where the acceleration $\ddot{y}_0$ is a function of time t; and term C is also a function of t and associated with $\ddot{y}_0$, $\dot{y}_0$, damping ratio β, and circular frequency ϑ. In general, for a random ground motion y_0, it is hard to determine an exact value of ξ so that the F in Equation 22 and in turn the function C in Equation 21 can be determined except a numerical solution is done for a specific earthquake motion y_0 with given values of β, ϑ and ξ.

When the upper limit frequency ϑ is assumed quite large or infinite as in the Arias derivation, the effect of term C on the energy can be ignored. In fact, the function F in Equation 22 has bound for any $0<\xi<t$ and large ϑ so that C in Equation 21 tends to zero. Thus, if the upper limit frequency ϑ is assumed quite large or infinite, the expression in the square brackets approaches unity so that Equation 23 can be further simplified to,

$$I_d=\frac{1}{\beta}\int_0^{t_d}\ddot{y}_0^2dt=C_d(\beta)\int_0^{t_d}\ddot{y}_0^2dt \quad (24)$$

which is a simplified earthquake intensity defined per unit mass with coefficient C_d.

Discussion on Earthquake Intensity

To see the difference between Equations 4 and 24, the two curves corresponding of coefficients with constant π/2 are drawn in Figure 1. It is observed that, the two intensity coefficients have the same value of 1 only when $\beta = 1$. When damping value β decreases, the difference of the two coefficients increase significantly. Particularly, when β approaches zero, coefficient C_d in Equation 24 tends to infinity, while Arias coefficient C_a in Equation 4 to π/2. Considering the C_a variation as indicated in Figure 1, the structural damping value β does not affect the intensity value significantly (Arias 1970). For instance, when damping ratio β varies from 0 to 0.2, the *Ca* value varies from 1.571 to 1.398 and the relative difference is 11%. This observation leads to a conclusion that selecting zero as a standardized β value for engineering structures does not depart too much the calculated intensity value. But this might violate the intention of using damping β to quantify structural damage as expected from the assumption. If $\beta = 0$, the dissipated energy E_d in the right term on the left-hand side of Equation 3 should be zero, but the earthquake input energy in the right term of Equation 3 is not equal to zero. This violates the condition of energy balance because the input earthquake energy is actually applied in the derivation, the value $C_a(0)$ of π/2 from Equation 4 or 5 might not represent the actual intensity behaviour due to damping energy. This issue had been noticed and explained that Equation 4 should be interpreted from a viewpoint of limit concept instead of an expression from directly setting $\beta = 0$ (Arias 1970). On the other hand, the new derived Equation 24 does not involve other mathematical assumptions as applied in the Arias derivation. The intensity coefficient $C_d(0)$ tends to ∞ but not π/2. This indicates that the energy-based earthquake intensity is infinitive, and the physical meaning is that the input energy cannot be balanced by the zero damping energy. When $\beta = 0$, the input energy should be balanced by kinetic energy plus strain energy, and no energy dissipation occurs in the structural system. It is also interesting to note from Figure 1 that the Arias intensity at zero damping level corresponds to the intensity value derived in this paper for damping value $\beta = 2/\pi = 0.637$.

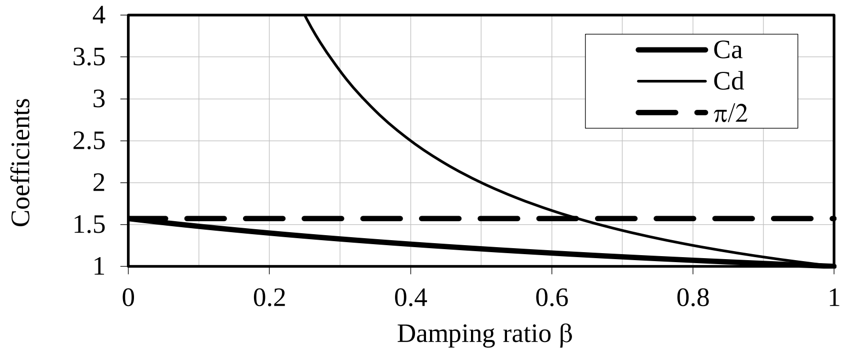

Figure 1. Comparison of intensity coefficients.

APPLICATION OF EARTHQUAKE INTENSITY

Energy Response with Zero Damping

To check the effect of damping ratio on the energy response, a dynamic analysis is conducted for the SDOF system without damping. The intention is to verify the energy behaviour for the situation when $\beta =$ 0 as the case for conventional Arias intensity. After transient time-history analysis for the SDOF system, the displacement and energy time histories are depicted in Figure 2. The strain energy response E_s in Figure 3 is not zero. It is seen that the damping energy E_d is zero throughout the loading process, and the input energy E_i is equal to the summation of kinetic energy E_k and strain energy E_s. This indicates that when $\beta = 0$, the basic assumption for Equation 4 is not correct because the energy balance condition $E_i = E_d$ is not satisfied for the derivation. Therefore, if the dissipated energy with a damping factor is used to represent the destructiveness or damage of structures, the damping ratio of β should not be set to zero.

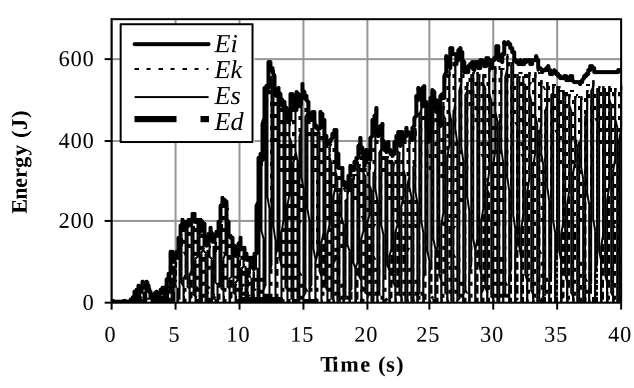

Figure 2. Energy response of elastic SDOF system with $\beta = 0$.

Energy Response of Seismic Sensors

Seismic instrumentation is important in the earthquake and structure engineering community. A seismic sensor consists of a suspended mass that moves relatively to the rigid instrument frame. The parameters for the sensors are assumed for the current study and the values are shown in Table 1. A seismometer has a relatively large mass (1 kg) and small stiffness (0.001 N/m) so that a small frequency 0.00503 Hz is achieved. High damping value is designed for seismic sensors to avoid phase distortion and β = 0.7 is normally chosen (Clough and Penzien 2003, Thomson 1981). It is interested to note from Equation 24 that the damping ratio β of 2/π (=0.63662 shown in Table 1) is close to 0.7 for seismic instruments. Therefore, the damping value β of 2/π is used in the analysis to obtain the sensor's response to base excitations. It is noted that a sensor system must behave elastically without any stiffness degradation and such a system is applied in the derivation of energy-based earthquake intensity. After performing transient analysis for the seismometer, the displacement response and the corresponding energy response are shown in Figure 3.

Table 1: Sensor's parameters for the transient analyses.

Sensor	m_2 (kg)	k_2 (N/m)	f (Hz)	β	c_2 (kg·m/s)	f_m (Hz)
Seismometer	1	0.001	0.00503	0.63662	0.04026	>0.015

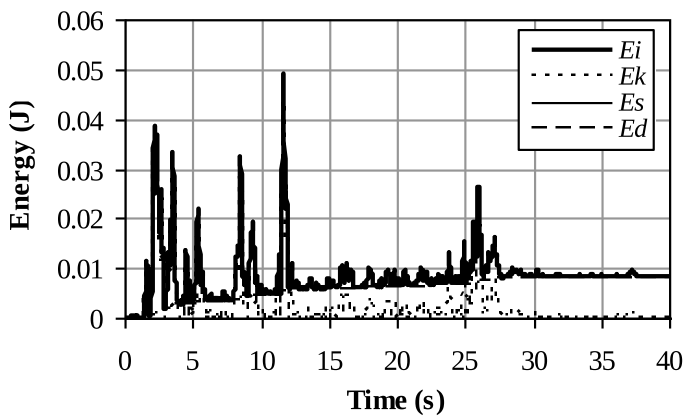

Figure 3. Energy response of seismometer with $\beta = 2/\pi$.

Note from Figure 3 that the peaks and valleys of the energy response are much more significant, and the sensor is more sensitive to the base excitation. Figure 3 indicates that the strain energy E_s is nearly zero and the level of damping energy E_d is low at about the time at which the maximum input energy E_i is attained. The kinetic energy E_k plays a significant role in this domain. However, when the base loading approaches the end, the kinetic energy E_k disappears, and the input energy E_i is balanced by the damping energy E_d alone

Intensity of Cumulative Energy

In accordance with the energy-based earthquake intensity expressed in Equation 4 or 24, a cumulative energy of the acceleration time series is defined by ASCE 4-16 standard (ASCE 2016):

$$I_{ASCE} = \int_0^{t_d} \ddot{y}_0^2 dt \tag{25}$$

which does not include the term of damping but it corresponds to $\beta = 1$ either in Equation 4 or 24. Equation 25 is used to determine the strong earthquake duration t_m corresponding to the cumulative energy to rise from 5% to 75% of total energy. The duration t_m is applied to calculate to one-side power spectral density defined by the Fourier amplitude of the earthquake acceleration time series (ASCE 2016).

In using actual recorded or artificial earthquake motion, structural analysts or engineers may face the problem of determining the length of strong motion traces that should be applied to enforce the superstructures. A common way for estimating strong motion duration is based on the energy-based earthquake intensity, and the duration t_m is defined as (Trifunac and Brady 1975):

$$t_m = t_{0.95} - t_{0.05} \tag{26}$$

where $t_{0.95}$ and $t_{0.05}$ are respectively the times at which 95% and 5% of the energy predicted by the Arias intensity. Equation (26) shows that in total, 90% of cumulative energy is applied to determine the strong motion duration. However, many records include a long tail of base motion with small amplitude up to the end of the motion, and this tail motion contributes much of energy to the total accumulated energy. Thus, the ASCE 4-16 standard (ASCE 2016) applies a 70% rule to quantify the strong motion duration:

$$t_m = t_{0.75} - t_{0.05} \tag{27}$$

in which $t_{0.75}$ is the time at which 75% of energy is reached using the standardized intensity in Equation (29). In applying the strong motion duration t_m, the total duration should include a rise time of $t_m/7$ and a decay time of $5t_m/7$ to obtain an enveloped duration for structural analysis (Salmon and Kennedy 1992).

To illustrate the characteristics of the strong motion durations defined above, the 1940 El Centro ground motion with parameters given in Table 1 is applied for the analysis. The horizontal acceleration of Impvall/I-ElC270 presented in the reference (Liu and Lu 2010). Upon using Equation 25 to the three acceleration time histories, the computation results are depicted in Figure 3.

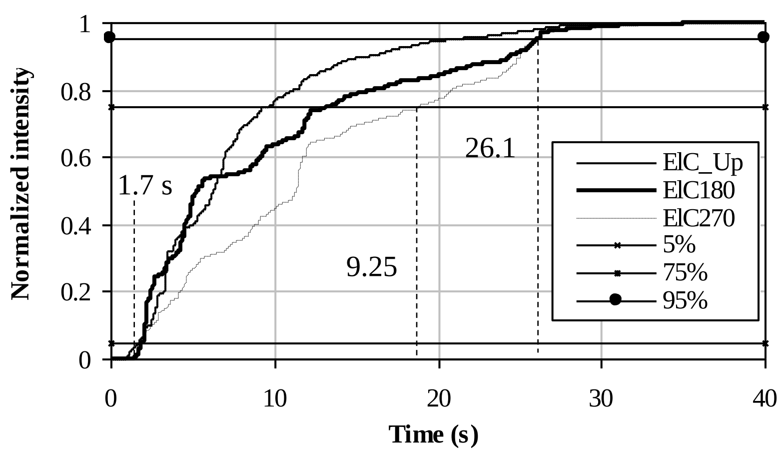

Figure 3. Energy response and strong earthquake time duration.

It is observed from Figure 3 that the vertical acceleration is first to reach the 5% input energy level at time 1.7 s. If the criterion in Equation 26 is used, the vertical acceleration attaints its 95% energy at about time 20 s, and then the other two accelerations reach their 95% energy level at time 26.1 sec. The effective strong motion duration is 24.4 sec when the horizontal accelerations are taken as the reference. Note that for determining the upper bound time of duration, the latest time history that attains its upper bound of energy criterion should be employed. Also, because the PGA of horizontal acceleration is greater than the PGA of vertical acceleration, the horizontal base excitation will cause a response from structures that is more severe than the vertical base excitation. Thus, it is reasonable to use horizontal time history as a reference to determine the strong motion duration. The time duration of 24.4 sec can contain almost all strong ground motion for structural analysis.

When the ASCE criterion in Equation 27 is used, it is seen that the vertical acceleration still attaints its 75% energy level at about 10 sec first, and then the ElC180, and at last the ElC270 acceleration reaches the energy level at 9.25 s (this is less than 10 s). The effective strong motion duration t_m is 9.08 sec when the ElC270 acceleration is taken as the reference. If the rise time 1/7 t_m and decay time 5/7 t_m are considered, the total time duration is about 17 sec, which is less than 24.4 sec from 95% criterion. Note that time duration of 17 sec can also contain almost all the strong ground motion for structural analysis, but the higher peaks of Impvall/I-ElC180 between 20 sec and 30 sec are lost. Thus, using 95% criterion to determine time history duration is on the conservative side for Fourier analysis. An investigation on predicting strong motion duration can be found in the literature to account for site and near-source influences based on Arias intensity (Kempton JJ, Stewart 2006).

CONCLUSION

It is shown from the analysis results that the derived intensity satisfies the energy-balanced condition. For an elastic structural system without damping, the earthquake input energy is transformed into kinetic energy plus strain energy of the structure, and the input energy would never be balanced. The conventional standardized intensity does not correspond to zero damping but to damping ratio equal to $2/\pi$ (≈ 0.7), which is a damping value commonly applied in the design of seismic sensors. Using energy-balanced intensity can provide reasonable insights to earthquake strength, rational information for assessing seismic margin, and designing warning systems for shutting reactors down. In using the energy-based earthquake intensity to identify the time duration of strong motion, as the 75% or 95% ratio is used, the value of damping ratio does not affect the estimate of the time duration of strong earthquake motion.

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