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A TAIL-ORIENTED MULTI-NORMAL MODEL FOR PARTIALLY CORRELATED SEISMIC-INDUCED FAILURE PROBABILITIES

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ABSTRACT

In seismic probabilistic risk assessment (SPRA) models, earthquake-induced failure probabilities should account for the degree of correlation due to the common-cause nature of earthquake-induced failures when quantifying the accident sequence logic tree. Since many seismic-induced failures are due to shaking effects in response to the same ground motions, there is correlation or dependence between these failures and their probabilities. Since the effects of shaking are propagated to equipment with varying configurations located at different floors or locations through potentially different buildings and foundations, this correlation is often only partial. Current state of practice typically represents this partial correlation in the SPRA model as one of either perfect correlation or full independence, depending on which approximation is more applicable. Methods and techniques to incorporate the modelling of partial correlation in SPRA models have received increasing attention in recent years. NUREG/CR-7237 (2017) presents a review and comparison of multiple methods and recommended the Separation of Independent and Common Variables (SICV) approach. A few recent SPRAs explicitly modelled partially correlated failures for structures or components with governing contributions to the SPRA model outcome (e.g., Talaat and Kennedy, 2019).

This paper presents a mathematical formulation and implementation for the tail-oriented multi-normal model (TMM) method, an enhanced SICV approach-based partial correlation modelling technique. The TMM method is superior to other techniques in that it uses an efficient closed-form formulation to compute the seismic fragilities for partially correlated failures. The closed form solution assumes that lognormal probability distribution is a valid representation of the conditional probabilities of seismic-induced failures, which is commonly accepted in SPRA models. The performance of the TMM method compares favourably to that of the SICV implementation in NUREG/CR-7237.

Techniques to model partial correlation in plant response logic trees and quantification, and their implementation, represent a technological challenge of high importance to multi-unit PRA (MUPRA) models. A technique is identified to efficiently integrate TMM-generated fragilities into MUPRA models.

INTRODUCTION

Seismic Probabilistic Risk Assessment (SPRA) models of nuclear power plants (NPPs) are composed of three major components: probabilistic seismic hazard analysis (PSHA), seismic fragility, and plant response logic models. The PSHA model determines annual occurrence rates for ground motion intensities, e.g., peak ground accelerations (PGAs). The fragility analysis of plant structures, systems, and components (SSCs) determines their conditional probabilities of failure given ground motion intensity by combining the probability distributions of the SSC strength and the corresponding seismic demand. The plant response logic model develops the accident sequences that lead to safe or unsafe shutdown and identifies minimum cutsets of SSCs that lead to "failure." The SPRA quantification code combines the SSC fragilities, cutsets, and seismic occurrence rates to calculate risk metrics such as core damage frequency (CDF).

The risk quantification code treats fragilities as independent failure events and uses Boolean math accordingly to calculate probabilities of union or joint failures. Plant logic model development recognizes that failure events of some SSCs are strongly correlated, such as co-located identical components that are much more likely to either all fail or all survive the same earthquake shaking effects. In common SPRA practice, strongly correlated SSCs are all represented by one fragility function, i.e., they are considered to be perfectly correlated. The remaining SSC failures are treated in the logic model as fully independent. This binary treatment of correlation based on informed judgment has been historically accepted based on a common recognition that (1) a systemic inclusion of partial correlation was not technologically feasible and (2) a more refined treatment of correlation is not expected to qualitatively change the outcome given the large number of NPP SSCs. Salmon and Whittaker (2015) presented a parametric study of the effect of partial correlation on the combined seismic fragility of two failure modes and its significance, which identifies situations where the latter premise is questionable.

Notwithstanding the scarce practical application, several efforts were spent in the literature to characterize partial correlation and their effect on seismic fragilities. Bohn and Lambright (1990) developed qualitative rules for assigning correlation coefficients to the seismic responses of SSCs. Reed, et al. (1985) proposed a technique that performs a Separation of Independent and Common Variables (SICV) analysis to develop a "composite" fragility curve for a cutset of partially correlated SSCs with known correlation coefficients.[1] The Reed, et al. (1985) method was implemented in NUREG/CR-7237 (2017) using double Latin Hypercube Sampling (LHS). Several other techniques were proposed by others, which are reviewed in NUREG/CR-7237, the simpler and more recognizable among which is the split-fraction method. NUREG/CR-7237 recommended the Reed, et al. (1985) method and outlined a proposed procedure to extend it to multi-layered partial correlation (e.g., Components A, B, C have partial correlation, and then Components A and B have additional pair-wise correlation independent of C, etc.) using multi-tier LHS. The method is mathematically elegant but numerically complex to implement, especially inside existing SPRA quantification codes.

The shift in the nuclear industry towards multi-unit sites, some with six or more identical reactor units, has outpaced the development of PRA technology, whose tools and guidance were established when single-unit (SU) sites were the norm or multiple units on a few sites were of different vintage and construction. Recent recognition that multi-unit PRA (MUPRA) results may differ significantly than traditional SU treatment has increased the need for a systemic treatment of partial correlation between seismic fragilities that may be significant risk contributors. In recent years, several industry organizations have started developing guidance focused on MUPRA, e.g., Electric Power Research Institute (EPRI) 3002020765 (2021) and several International Atomic Energy Agency (IAEA) publications, e.g., Safety Report No. 96 (2019). Anup, et al. (2021) presents partial correlation determination guidance developed in EPRI 3002020765. An efficient, methodical modelling technique for SSC partial correlations is of higher importance for MUPRA than SUPRA and motivates the development of the proposed TMM method.

USEFUL TECHNICAL BACKGROUND

This section presents selected elements of set and probability theory that are useful for developing the TMM method. Figure 1 shows a sample space S with probabilities of failure of three identical SSCs of ID 'A' at any ground motion intensity. Figure 1.a shows the set theory representation when the fragilities of A are fully independent. Figure 1.b shows the case where these fragilities have strong positive correlation. The

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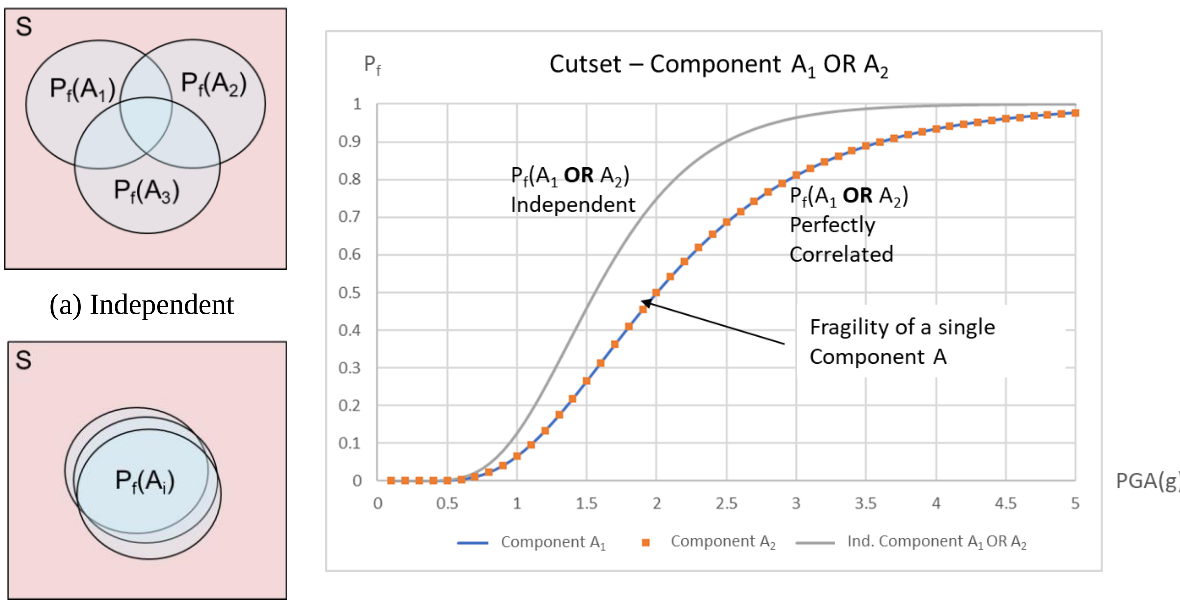
[1] This paper uses the term "composite fragility" to refer to a single fragility curve for a combination of SSC failures.

probabilities for concurrent (3 out 3) failures and union (1 or more out of 3) failures of SSC A when fully independent are as shown in Equations 1 and 2:

$$P_f(A_1 \cap A_2 \cap A_3) = P_f(A_1)\, P_f(A_1)\, P_f(A_1) = P_f(A)^3 \quad (1)$$

$$P_f(A_1 \cup A_2 \cup A_3) = 1 - [1 - P_f(A)]^3 \quad (2)$$

For positive partial correlation between these SSCs, it is clear from Figure 1 than the intersection probability is larger than that determined using Equation 1 while the opposite is true for the union probability and Equation 2. Negative correlation is rare in seismic fragility. Perfect correlation would collapse all three $P_f(A)$ domains in one outcome for any combination of SSC A failures. Figure 1c shows the composite seismic fragility curves of a cutset composed of the union of either SSCs A_1 or A_2 failure for the two extreme cases of full independence and perfect positive correlation. The composite fragility for the perfectly correlated case is simply the same as the fragility of SSC A. For the fully independent case, it is generated by following the example in Equation 2 at discrete PGAs.



(a) Independent

(b) Partially Correlated

(c) Single-SSC and Composite Fragility Curves

Figure 1. Set Theory Representation and Seismic Fragilities of Independent and Correlated Failures.

The examples presented above used identical SSCs with the same seismic fragility for ease of demonstration. Without loss of generality, the preceding discussion can be extended to partially correlated SSCs with dissimilar fragility parameters. Finally, Baye's rule presents a powerful link between conditional, joint, and marginal failure probabilities (Equation 3a):

$$P_f(A_1 \mid A_2) = P_f(A_1 \cap A_2) \,/\, P_f(A_2) \quad (3a)$$

and can be rearranged as:

$$P_f(A_1 \cap A_2) = P_f(A_1 \mid A_2) \times P_f(A_2) \quad (3b)$$

where:

$P_f(A_1 \mid A_2) = P_f(A_1)$ if fully independent

$P_f(A_1 \mid A_2) = 1.0$ if perfectly correlated

$P_f(A_1 \mid A_2) < P_f(A_1 \mid A_2) < 1.0$ if partially correlated (positive)

For independent failures, knowing that one SSC has failed provides no additional knowledge about the failure probability of the other SSC(s). For perfectly correlated failures, it informs that the other SSC(s) have certainly failed. For partially correlated SSCs, informs that failure of the other SSC(s) (conditional probability) is more likely than it was in the absence of that knowledge (marginal probability).

CHALLENGES WITH MODELLING PARTIAL CORRELATION IN SPRA

While the mathematical framework for incorporating partial correlation between seismic fragilities in an SPRA is relatively straightforward, several challenges make its real-world application impractical. Four significant technical challenges are discussed here:

- Technical basis for determination of correlation coefficients
- Efficient calculation of partially correlated fragilities for all relevant cutsets
- Streamlined incorporation of results in logic model using existing software
- Lack of a default "conservative" modelling decision

The first challenge is one where significant literature and proposed methods exist. It is important to recognize that the meaning and determination basis of correlation coefficients depends on how the partially correlated fragilities will be mathematically modelled, which is linked to the second challenge. EPRI 3002020765 (2021) presents guidance on the development of correlation coefficients for use with the SICV approach of modelling partial correlation. EPRI 3002010663 (2017) presents an alternative definition of correlation coefficients whose development guidance follows a different process. Reed, et al. (1985) discusses two alternate definitions. This paper follows the definition of fragility correlation adopted in EPRI 3002020765 (2021) and presents more specific recommendations applicable to the TMM technique.

The second challenge is the major motivation and focus of this paper. The partially correlated fragility development techniques presented in NUREG/CR-7237 (2017) are either efficient but approximate (e.g., split fraction method) or rigorous but computationally demanding to methodically implement in a complex NPP SPRA model. NUREG/CR-7237 acknowledged this challenge and suggested that the recommended partial correlation modelling guidance be applied selectively for the SSCs in cutsets or accident sequences where partial correlation analysis "makes a difference." Determination of these cutsets would be based on a first-pass SPRA model quantification using state-of-practice correlation modelling (e.g., binary) and possibly on multiple rounds of iteration until the quantifications results converge. Talaat and Kennedy (2019) presented a case where identification of the partial correlation case to consider followed this approach and a composite fragility was developed for these partially correlated SSCs.

The third challenge is a consequence of the legacy of existing PRA logic modelling and quantification software. EPRI 3002020765 (2021) reported that, in order to develop and use the partially correlated fragilities, close interaction had to take place between the fragility analyst and the systems analyst to identify (1) which cases of 'n out of N' SSCs should the composite fragilities be developed for and (2) how to input this information into the quantification without rewriting the accident sequences in the logic model. EPRI 3002020765 (2021) identified two alternatives, each with nontrivial limitations.

- The first alternative was to compute a library of composite fragilities considering partial correlation for the different SSC failure combinations of interest to the SPRA, e.g., any two of N components failing, any three of N components failing, etc. This computation is performed outside of the SPRA quantification code. The composite fragility is then input to the code. This entails that the different combinations of SSC failures of interest be identified a-priori and modelled as distinct and mutually exclusive basic events in the logic tree, resulting in significant required revision to the logic model. It also requires continuous or close periodic coordination between the fragility analyst and the systems analyst.

- The second alternative is to add logic tree branches that represent a "perfectly correlated" fragility solution to an original tree which considers all failures to be independent, still compute the composite fragilities outside of the SPRA quantification code, and then back-calculate calibrated logic tree weights for the "perfectly correlated" and "fully independent" alternatives that recover the composite fragilities using the split-fraction approach. This alternative can be readily implemented in common SPRA logic software but involves approximation and proxy basic events that may mask risk insights.

The fourth challenge is the lack of a quick rule to simplify the modelling of partial correlation between SSCs, when it is significantly different from weak or strong, as either zero or one on a consistently conservative basis in lieu of explicitly modelling it. For example, simplifying partially correlated fragilities of redundant similar components as perfectly correlated is conservative while the opposite is true for components which are "in series" on a success path for the NPP for positive correlation. (The inverse of this statement is true for negative partial correlation, which is a rare occurrence.) For the complex logic trees of NPP accident sequences involving safety, protection, and mitigation systems, readily determining which simplification is conservative for each group of partially correlated SSCs can be time-consuming.

PROPOSED TMM FRAGILITY MODEL FOR PARTIALLY CORRELATED FAILURES

The proposed method builds upon the SICV concept. The fundamental difference between it and the Reed, et al. (1985) technique is that its implementation takes advantage of the normal distribution additive property, whereby the sum of independent, normally distributed random variables is normally distributed with an expected value and variance equal to the sum of expected values and variances, respectively, to efficiently develop a composite fragility for any combination of SSC failures. Seismic fragility curves of individual SSCs are represented by lognormal probability distributions, whereby the log-transform of the random variable (i.e., the seismic capacity, which is the ground motion intensity, e.g., PGA, at which the resulting seismic demand on the SSC reaches its capability to maintain its required function in the plant logic model) is normally distributed. The product of independent lognormally distributed random variables is lognormal with a median seismic capacity equal to the product of the medians and a logarithmic standard deviation equal to the square root of the sum of the squares (SRSS) of those of the constituent random variables. The lognormal distribution of an SSC seismic capacity with a known median and standard deviation can be perfectly expressed as the product of any independent lognormal random variables whose medians and standard deviations satisfy the conditions above. This allows the development of lognormal composite fragilities that use a closed-form formula to closely match the lower tail of the empirically derived composite fragility. The lower ~10% of a fragility function typically contributes about 50% or more of its seismic annual frequency of failure, and the lower half contributes typically about 90% or more.

Concept

The symbols A_m and β are used for the median seismic capacity and logarithmic standard deviation, respectively. The seismic fragilities of SSCs, represented using logarithmic probability distribution functions, are fully defined using these two parameters. The parameter β typically consists of the SRSS of several variability source contributions. These contributions are typically grouped into two terms representing epistemic uncertainty and aleatory randomness, β_u and β_r. They can also be divided into random variables that introduce variability into the SSC seismic demand and its capacity to withstand it, β_D and β_C. Similarly, for any group of N SSCs designated i = 1 to N that are partially correlated, their respective individual β_i parameters may be represented as the product of two parameters, $\beta_{1..N}$ and $\beta_{I,i}$, where:

- $\beta_{1..N}$ is the logarithmic standard deviation due to the common random variables
- $\beta_{I,i}$ is the logarithmic standard deviation due to the independent variables for the i^{th} SSC
- i = 1, 2, ..., N is the SSC index

Observe that it is sufficient to determine the common parameter $\beta_{1..N}$ and then use Equation 4.

$$(\beta_{I,i})^2 = \beta_i^2 - (\beta_{1..N})^2 \qquad \text{for all i} \tag{4}$$

Given the additive property of the normal distribution, the seismic capacities of these partially correlated SSCs may each be represented as the product of two lognormal random variables, $A_{1..N}$ and $A_{I,i}$ with the variability parameters $\beta_{1..N}$ and $\beta_{I,i}$, respectively, and corresponding median capacities. Each two median capacities, $A_{m,1..N}$ and $A_{m,I,i}$, may be arbitrarily assigned any two values whose product is equal to $A_{m,i}$. It is practical and logical to set $A_{m,1..N}$ = 1.0 and $A_{m,I,i} = A_{m,i}$. The random variable $A_{I,i}$ describes an SSC seismic fragility of the same median capacity of SSC_i and a reduced variability that represents only the independent random variables. The random variable $A_{1..N}$ is a mathematical construct: a lognormal probability distribution with a dummy median capacity equal to unity and a variability that represents the common random variables. These random N+1 variables are all fully independent of each other.

The random variables $A_{I,i}$ are fully independent, and corresponding composite seismic fragilities for any combination of $SSC_{i=1\text{-to-}N}$ failures can be computed following Equations 1 and 2 at discrete PGAs. These composite fragilities, referred to as Step-1 composite fragilities thereafter, represent the desired composite fragility only if the common random variable $A_{1..N}$ is deterministic and has a unit value. To account for the additional variability due to common sources, a Step-2 composite fragility is constructed by combining the Step-1 fragility with the lognormal probability distribution of the common random variable $A_{1..N}$. Prior to this combination, the Step-1 composite fragility is converted to an equivalent lognormal random variable $A_{I,j}$ using the TMM technique, where subscript j identifies the SSC failure combination. Since both $A_{I,j}$ and $A_{1..N}$ are lognormally distributed, the Step-2 composite fragility is computed in closed form by combining the two distribution parameters using the additive property of the normal distribution.

Implementation Steps

The TMM method implementation uses the following steps:

1. Separate the independent and common random variables.
2. Develop the Step-1 composite seismic fragility representing independent variables only.
3. Perform TMM calibration of the Step-1 lognormal fragility.
4. Develop the Step-2 composite fragility combining independent and common variables.
5. Perform additional post-processing (optional).

Separation of Independent and Common Variables

In a lognormal fragility model, the seismic capacity is the product of a number of lognormal random variables that govern the seismic demand and capacity of the SSC. The logarithmic standard deviations characterizing seismic fragilities are the combination of standard deviations due to multiple sources of randomness and uncertainty, collectively referred to as sources of variability. Through identifying these distinct sources, one can separate the sources of variability that are common to multiple SSCs from the sources that are individually applicable to each SSC and can be considered independent. Once the SICV is performed, a partial correlation coefficient for the seismic fragility can be computed explicitly. While this partial correlation coefficient is useful to report as a summary statistic, its value is not required for the implementation of the TMM method.

Variability sources can be classified into those related to either the seismic demand or capacity of an SSC. Variability in the seismic capacity is due to factors like material strength and ductility, equations for transforming material strengths into component strength (e.g., bending moment corresponding to yield), and period elongation and energy dissipation due to inelastic behaviour. Variability in the seismic demand is due to randomness in the ground motion input, uncertainty in the soil properties, variability in the dynamic properties and response of the structure that houses the SSC (if applicable), and variability in the dynamic properties and response of the SSC (e.g., frequency, damping, mode shapes, and phasing).

SICV can be performed by tracking each one of these variability sources (and sub-sources) and determining whether their effect is common or independent on the SSCs in question. For example, equipment installed using the same anchor model and design at the same time into the same concrete floor have common material strength variabilities and have independent variability in the anchorage strength equation model equal to the standard error in that model compared to laboratory tests. Section 3.2.5 in EPRI 3002020765 contains a detailed discussion of these sources and guidance on considerations applicable to when they are common or independent for SSC groups. Once the common and independent variables are identified, the common logarithmic standard deviation can be computed using Equation 5, and the $\beta_{I,i}$ terms can be computed using Equation 4.[2]

$$(\beta_{1\,..\,N})^2 = \Sigma(\beta_{\text{common variables for SSCs }1,2,..,N})^2 \tag{5}$$

While this process may sound tedious, it needs to be performed only for the dominant variability sources since the SRSS combination diminishes the influence of the less major sources on the outcome. An effective correlation coefficient can thus be explicitly determined using Equation 6a for any SSC pair:

$$\rho_{12} = (\beta_{12})^2 / \beta_1\beta_2 \tag{6a}$$

or,

$$(\beta_{12})^2 = \rho_{12}\beta_1\beta_2 \tag{6b}$$

where ρ_{12} is the correlation coefficient for this variability source in the two fragilities. Equation 6b can be used to determine the common logarithmic standard deviation for SSC pairs if the correlation coefficient is known. The SICV implementation in Reed, et al. (1985) uses Equation 6b. Equation 6b is applicable to any component of the variability due to individual sources and not only to the resultant. It can therefore be utilized as a building block in the systemic SICV process outlined above. When a variability source cannot be considered fully common or independent but itself involves partial correlation, or is the aggregate effect of several sources that could not be readily dissected into common and independent ones, its common β component can be computed using Equation 6b. Section 3.2.5 in EPRI 3002020765 presents a discussion of how to develop judgment- and analysis-based estimates of ρ for aggregated variability sources. The outcome of composite fragilities demonstrated limited sensitivity to precision in assigned ρ values. Meanwhile, the common β components due to other sources can be determined as described earlier, and all common components combined using Equation 5.

Development of Step-1 Composite Fragility – Independent Variables

The Step-1 composite fragility is developed using Boolean logic for the combination(s) of SSC failures that are required for the accident sequences in the SPRA logic model. Typical combinations of interest include "all SSCs in the partially correlated group fail" or "any SSC in the partially correlated group fails." Each failure combination 'j' has a corresponding Step-1 composite fragility. The Step-1 composite fragility uses only the distributions of the independent random variables $A_{I,i}$. It is derived empirically by discretizing the individual fragility functions and performing the Boolean math operations according to Equations 1 and 2 on the conditional probabilities at each discrete PGAs, e.g., using a compact spreadsheet macro or script. As discussed next, explicit output and storage of the empirical Step-1 fragilities may not be required.

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[2] For brevity, the implementation in this paper presents the typical case of the common variability $\beta_{1\,..\,N}$, being equal for all SSCs in the group. The formulation can be extended to unequal β_{12}, β_{13}, etc., as discussed in NUREG/CR-7237.

Tail-oriented Multi-normal Model Calibration

When convolved with the seismic hazard curve in SPRA quantification, the seismic risk contribution of a seismic fragility curve is dominated by its lower tail. Development of a lognormal equivalent of the Step-1 composite fragilities identifies the parameters of a distribution calibrated to fit the lower tail of the empirical Step-1 fragility. After the empirical Step-1 distribution is generated, fitting the two lognormal parameters can be performed using two points on the empirical fragility selected to produce an acceptable fit to the lower tail. It is recommended to select one point for the TMM calibration in the vicinity of the PGA that corresponds to 5% probability of failure ($A_{5\%,I,j}$) and a second point between ($A_{5\%,I,j}$) and ($A_{50\%,I,j}$) by interpolation along the empirical Step-1 fragility curve. These two values can then be used to calculate the calibrated lognormal distribution parameters according to Equation 7.

$$\beta_{I,j} = LN(A_{q1\%,I,j} / A_{q2\%,I,j}) / [\Phi^{-1}(q1\%) - \Phi^{-1}(q2\%)] \quad (7a)$$
$$A_{m,I,j} = A_{q1\%,I,j} \exp(\beta_{I,j}\, \Phi^{-1}(q1\%)) \quad (7b)$$

where Φ^{-1} is the inverse normal probability function and q1% and q2% are the Step-1 conditional probabilities of failure at the two points selected for the TMM calibration. Performing the TMM calibration in this manner requires outputting and examining the Step-1 composite fragility function and examining the quality of the TMM fit using alternative point selections. This process ensures a precise fit process but can become time consuming for a large SPRA model.

An efficient alternative to calibrate the TMM fragility parameters can be performed at the expense of potentially reduced precision. In this alternative, the q1% and q2% are pre-selected without examining the Step-1 fragility. To generically generate a robust lognormal fit of the lower tail, selected values of q1% and q2% should be near the upper and lower bounds of the PGAs of interest to the SPRA. In the examples presented in this paper, $A_{q1\%}$ and $A_{q2\%}$ were set to $A_{50\%}$ and $A_{1\%}$, respectively. An interpolation function was written to directly compute these parameters directly without explicitly calculating the Step-1 fragility. The validation examples compare the precision of the TMM fragilities calibrated following both approaches.

Development of Step-2 Composite Fragility – Independent and Common Variables

The Step-2 composite fragility combines the random variables $A_{I,j}$ (from Step-1) and $A_{1..N}$ into a single lognormal random variable A_j for each failure combination 'j'. Equation 8 identifies its parameters.

$$A_{m,j} = A_{m,I,j} \quad (8a)$$
$$(\beta_j)^2 = \beta_{I,j}^2 + (\beta_{1..N})^2 \quad (8b)$$

Additional Post-Processing and Optional Steps

Post-processing can be performed on the output generated in the previous steps to produce useful quantities that can be used to summarize, review, and/or further refine the developed fragilities. Such post-processing may include calculating an effective correlation coefficient for the SSC fragilities that represents the relative influences of the common and independent variables (Equation 6a) and splitting the variability parameter β into randomness and uncertainty components.

VALIDATION AND COMPARISON TO OTHER TECHNIQUES

Two validation examples are presented in this paper. Each example compares the composite fragilities of "1 or more out of N" and "N out of N" failure combinations of SSCs determined using three methods: the Monte Carlo Simulation (MCS) method described in Talaat and Kennedy (2019), the Reed and McCann method (Reed, et al., 1985) implemented using double LHS (RM-LHS) as in NUREG/CR-7237 with thirty samples, and the TMM method. The TMM solutions are shown for lognormal parameters calibrated using both (1) two points selected to fit the tail (TMM-F) and (2) two points pre-selected as discussed earlier (TMM-G). In assessing the validation results, recall that the lognormal distribution model used for the individual SSC fragilities is itself an analytical idealization that represents the conditional failure probability within acceptable engineering tolerance. The TMM method applies the same concept to develop an analytical representation of the composite fragilities of failure combinations of SSCs.

Validation Example 1

Two SSCs, A and B, have distinct fragility parameters and are correlated with $\rho = 0.42$. Table 1 lists these SSC fragility parameters. Figures 2a and 2b compare the composite fragility curves for the two failure combinations using the three methods. Composite fragilities for full independence and perfect correlation are shown for reference. Insets in each figure show a close-up of the curves at probabilities lower than 0.05. Intermediate results for step-by-step implementation could not be shown due to space limitation.

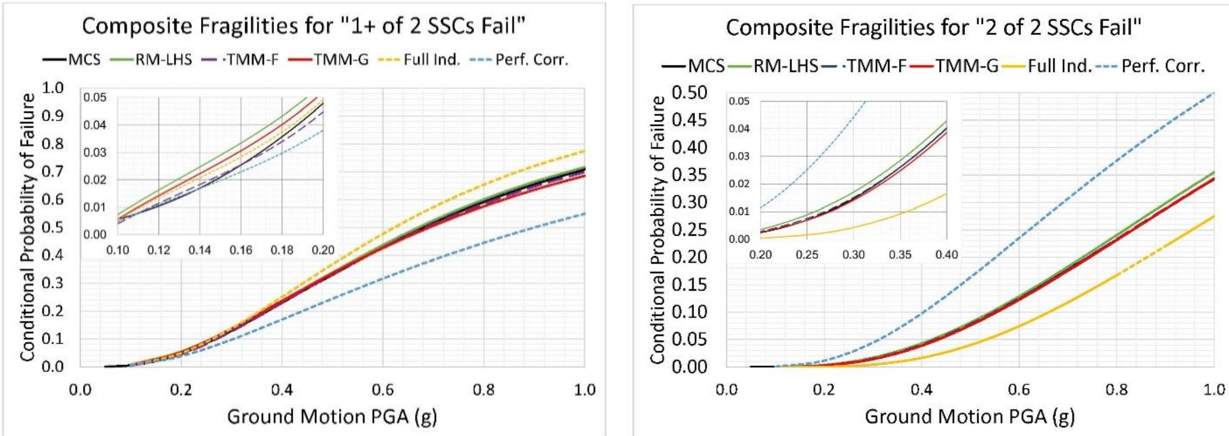
Figure 2a shows that the "union" composite fragility curves computed using the three methods are generally comparable. The explicitly fit TMM fragility (TMM-F) shows the closest match to the MCS solution, and both curves are bound between the extreme-case curves (with minor precision exceptions at low probabilities). At conditional probabilities of about 0.15 and lower, the TMM-G and RM-LHS solutions compute somewhat higher probabilities than the MCS solution and slightly exceed the upper bound set by the fully independent case. These discrepancies are likely acceptable to the SPRA accuracy if they have non-significant risk contributions. At probabilities higher than 0.15, the RM-LHS fragility is consistently slightly above the MCS solution, the TMM-F solution shows a close match or slight reduction, and the TMM-G solution is slightly higher on one end and somewhat lower on the other end. Discrepancies at failure probabilities higher than about 0.5 are of typically low significance to SPRA quantification. Overall, the TMM-F solution shows the best match to the MCS solution. The more efficient TMM-G solution shows an average match of lower precision that should be acceptable for other-than highly risk significant SSCs.

Figure 2b shows that the "intersection" composite fragility curves computed using the three methods are generally comparable and almost indistinguishable at low probabilities. As the probabilities increase, the RM-LHS composite fragility shows small but consistent increases compared to the other methods. Both the explicitly and generically fit TMM fragilities very closely match the MCS fragility. All three methods compute composite fragilities that are well within the extreme bounds. The following general observations can be made after comparing the results in Figures 2a and 2b:

- Failure probabilities with ρ approaching 0.5 are clearly closer to fully independent than perfectly correlated fragilities, especially at low probabilities. Thus, the effect of partially correlated capacities on SPRA outcome diminishes quickly away from perfect correlation towards full independence.
- The explicitly and generically fit TMM-F and TMM-G solutions show limited differences.
- The composite fragility for failure unions (Figure 2a) appears to be more sensitive to the correlation modelling method and implementation details than intersections (Figure 2b).
- A split-fraction method can be used to model the partially correlated composite fragility as a weighted sum of the two extreme cases with calibrated PGA-dependent fractions to reproduce the conditional probabilities of failures with values between the two extremes.

Table 1: SSC Fragility Parameters for Validation Examples.

SSC ID	A	B	C
A_m(g)	0.9	1.0	1.48
β	0.85	0.71	0.35



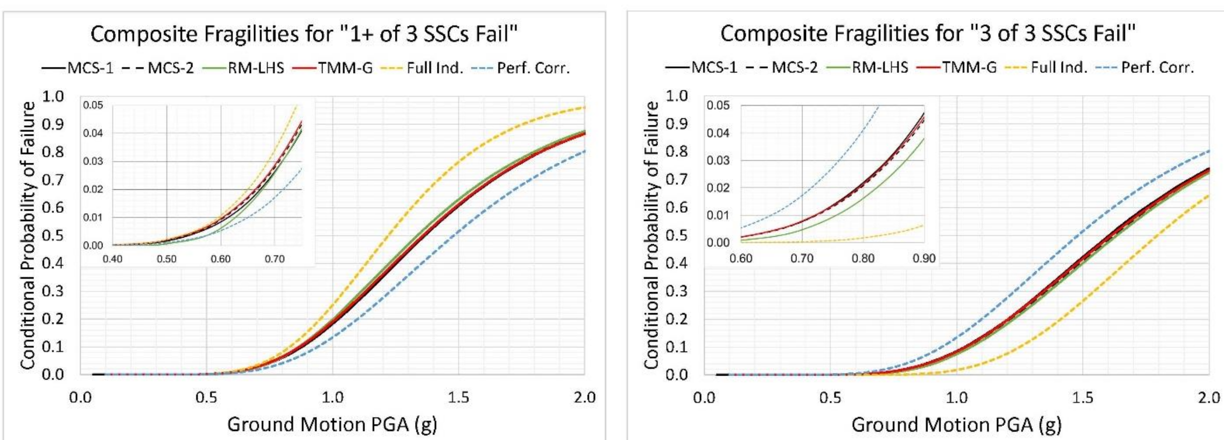
(a) 1 or more SSCs Fail

(b) All SSCs Fail

Figure 2. Composite Fragility Curves for Validation Example 1.

Validation Example 2

The effect of partial correlation between three strongly correlated SSCs is analysed. The fragility parameters of these three SSCs correspond to component C in Table 1. In a traditional SPRA, they would be considered perfectly correlated. In this example, they are modelled with ρ = 0.9. Figures 3a and 3b compare the composite fragility curves for the two failure combinations using the three methods and show the solutions for the full independence and perfect correlation solutions for reference. Two MCS simulations were performed to contextualize the observed differences within the MCS numerical precision. The MMT results are only shown for the TMM-G implementation as discussed below.



(a) 1 or more SSCs Fail

(b) All SSCs Fail

Figure 3. Composite Fragility Curves for Validation Example 2.

Figure 3a shows that the "union" composite fragility curves computed using the two MCS runs are almost identical, and that the fragility computed using the TMM-G method essentially matches the MCS-2 solution. Given this very close match, explicitly fitting and plotting the TMM-F solution was not needed and was excluded for clarity. The RM-LHS solution is close to the three other fragilities but consistently higher at conditional probabilities of about 0.10 and higher. All the computed composite fragilities are within the two extreme bounds. Figure 3b shows that the "intersection" composite fragility curves computed using the two MCS runs are slightly different, and that the TMM-G fragility curve is in between the two. The RM-LHS solution is practically a close match to the three other fragilities while consistently slightly lower. All the computed composite fragilities are within the two extreme bounds. Similar to that observed in Example 1, the composite fragility for failure unions (Figure 3a) appears to be more sensitive to the correlation modelling method and implementation details than intersections (Figure 3b).

Despite the strong correlation between seismic capacities, the conditional probabilities are not much closer to the fully correlated solution than the fully independent one, especially at low probabilities of failure. This is an important observation that underscores the value of treating partial correlation using a practical mathematical solution instead of relying on intuition, e.g., assigning split fractions by judgment.

INTEGRATION OF TMM-GENERATED FRAGILITIES IN SPRA AND MUPRA MODELS

Integration of the composite fragilities representing combinations of SSC failures into existing SPRA models represents a challenge, as discussed earlier. Current applications using legacy SPRA logic models and quantification codes have resorted either to (1) solving parts of the logic tree that represent risk-significant partially correlated fragilities outside the SPRA code and feeding the resulting fragilities as input to a modified logic model (e.g., Talaat and Kennedy (2019) and NUREG/CR-7237 guidance) or to (2) systematically modifying affected branches on the logic tree to add "fully independent" and "perfectly correlated" alternatives of the basic event whose fault trees the SPRA code can solve and then combine using split-fractions calibrated for failure combination by the fragility analyst (e.g., EPRI 3002020765).

Jung, et al. (2020) recently developed a methodology to systematically represent partially correlated seismic fragilities in SPRA logic models for MUPRA. This methodology modifies the fault trees to replace the basic events of partially correlated failures in multiple units by an equivalent event set of proxy common-cause failure (CCF) events. The equivalent CCF events represent all the possible failure combinations of the group of correlated SSCs. For example, for a group of three SSCs, the three basic events of SSC A, B, and C failures are replaced as shown in Equation 9 for Event A.

$$A = C_A + C_{AB} + C_{AC} + C_{ABC} \tag{9}$$

where Events C_{ijk} are CCF events of the SSC failures indexed in each event, i.e., C_A is "failure of only A) while C_{AB} is "concurrent failure of only A and B" and so on. It can be readily seen that the CCF events are collectively exhaustive, such that they fully replace the basic events, and mutually exclusive, such that partial correlation between them need not be explicitly considered in the quantification code. The partial correlation is systemically considered in mapping the conditional probabilities of the basic events (e.g., failures of SSCs A, B, and C) to those of the replacement CCF events. Jung, et al. (2020) describes the one-to-one transformation between the conditional probabilities of the possible SSC failure combinations, P, to the conditional probabilities of the replacement CCF events, Q. Once the probabilities Q are determined, the SPRA quantification of the SPRA logic tree proceeds using existing quantification codes with updated cutsets that use the CCF events in lieu of the replaced basic event.

The methodology proposed in Jung, et al. (2020) presents an elegant and efficient solution to incorporating partially correlated seismic fragilities in SPRA logic models and quantification that is not limited to MUPRA. The execution of this method requires seismic fragility curves for all possible failure combinations within the partially correlated SSCs of interest to the SPRA model. The TMM method efficiently develops these composite seismic fragility curves for input to the CCF transformation equations from P to Q probabilities. Integrating the TMM method with the Jung, et al. (2020) methodology produces an efficient SPRA modelling and quantification technology that explicitly incorporates partially correlated failures using commonly used computer codes and tools.

CONCLUSION

This paper presents a novel technique to develop composite seismic fragilities for any combination of partially correlated SSC failures. The technique uses the additive property of multi-normal random variables to develop an efficient closed form solution calibrated to the lower tail of the subject fragility. Two alternatives were presented for performing the TMM calibration, which offer a trade-off between precision and efficiency to accommodate both risk-significant fragilities and production-scale application. Benchmark validation examples against composite fragilities computed using MCS and the Reed, et al. (1985) methods demonstrate the precision and efficiency of the TMM method using both alternatives. The benchmark validation also indicates that probabilities of intersection failure combinations are less sensitive to the partial correlation modelling technique than those of failure union combinations, and that small reductions in correlation from being perfect strongly reduces the effect of correlation on the probabilities. Further research can be performed using a broad range of representative examples for SUPRA and MUPRA to identify constraints on the TMM method applicability and develop refined guidance for its production-scale calibration under different conditions (e.g., strong correlation vs. moderate correlation and union vs. intersection failure combinations). The paper finally demonstrates that TMM-derived fragilities can be readily integrated into recently developed MUPRA modelling methods.

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