

# Nonlinear, Inelastic ESSI Analysis

## SMiRT26 Tutorial II

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# Outline

Introduction  
    Motivation

Inelastic ESSI Analysis  
    Analysis Phases  
    Modeling and Simulation Components

Modeling and Simulation  
    ESSI Modeling, Calibrations  
    ESSI Simulation, Parameters

Summary

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# Motivation

Improve modeling and simulation for infrastructure objects

Reduction of modeling uncertainty

Choice of analysis level of sophistication

Goal: Predict and Inform

Engineer needs to know!

# Dedication

Robert P. Kennedy, 1939-2018



"Response of a soil structure system is nonlinear, and I would really like to know what that response is!"

Nebojša Orbović, 1962-2021



"As an engineer, I have to know what are response sensitivities to modeling choices and model parameters."

# Outline

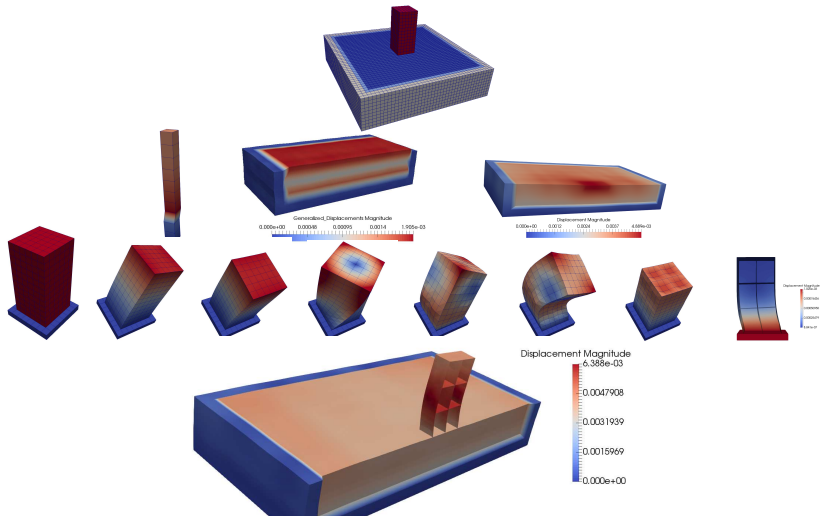
Introduction  
Motivation

**Inelastic ESSI Analysis**  
**Analysis Phases**  
Modeling and Simulation Components

Modeling and Simulation  
ESSI Modeling, Calibrations  
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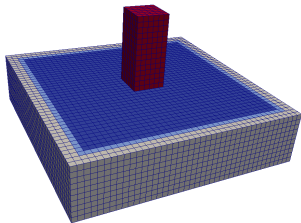
Summary

# Analysis Modeling Phases



# ESSI Modeling Phases

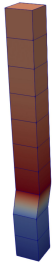
- Account for all model physical components
- Solids, structures and fluids
- Elastic, inelastic materials
- Static loads
- Dynamic loads
- Response quantities
- Engineer/Analyst builds confidence in analysis
- No surprises and no "reliance" on good luck!





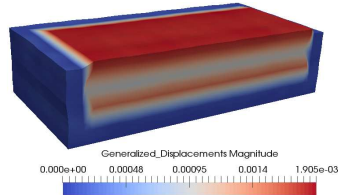
# ESSI Modeling Phases

- 1D, 1C free field response
- Linear elastic material
- Inelastic material



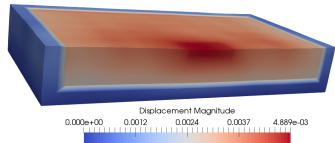
# ESSI Modeling Phases

- 3D, 1C free field response
- Linear elastic material
- Inelastic material



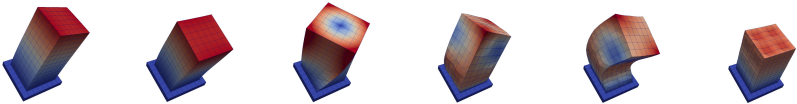
# ESSI Modeling Phases

- 3D, 1C, part of SSI response
- 3D, 1C, add SSI components
- Linear elastic material
- Inelastic material



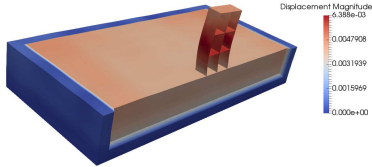
# ESSI Modeling Phases

- Eigenvalue analysis



# ESSI Modeling Phases

- Synthesis: full ESSI model



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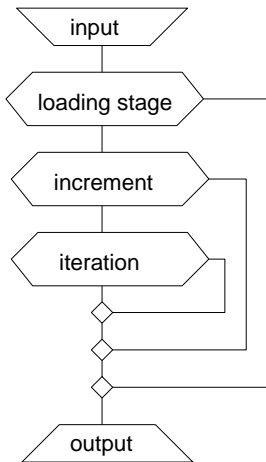
Modeling and Simulation  
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Summary

# ESSI Modeling

- ESSI inelastic modeling, mechanics
  - Soil
  - Interfaces/Joints/Contacts
  - Foundation
  - Superstructure
- ESSI inelastic simulation, numerics
  - Constitutive integrations, explicit/implicit
  - FEM static solution advancement
  - FEM dynamic solution advancement, time stepping

# ESSI Simulation





# Material Models

- Linear elastic, all materials
- Nonlinear elastic, solids
- Soil, solids/3D, dry, saturated and partially saturated)
- Rock, solids/3D, dry, saturated and partially saturated)
- Contact, soft and hard, gap, 2-node/3D, dry and saturated
- Base isolator and dissipator 2-node/3D
- Concrete, solids/3D, wall/2D, beam/1D
- Steel, beam/1D and solid/3D

# Material Models for Soil and Rock, Total $\sigma$

Fully saturated clay and rock with very low permeability, dry solid bricks

- vonMises
- vonMisesArmstrongFrederick
- vonMisesMultipleYieldSurface
- vonMisesMultipleYieldSurfaceGoverGmax

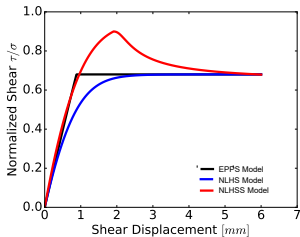
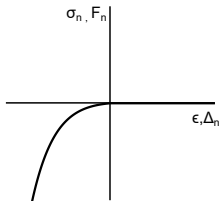
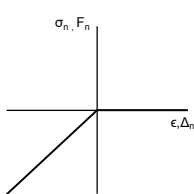
# Material Models for Soil and Rock, Effective $\sigma$

Dry and/or partially saturated/unsaturated and/or fully saturated clay and rock with any permeability, dry bricks and u-p-U bricks

- DruckerPrager
- DruckerPragerArmstrongFrederickLE
- DruckerPragerArmstrongFrederickNE
- DruckerPragerNonAssociateLinearHardening
- DruckerPragerNonAssociateArmstrongFrederick
- roundedMohrCoulomb
- CamClay
- DruckerPragerMultipleYieldSurface
- DruckerPragerMultipleYieldSurfaceGoverGmax
- RoundedMohrCoulombMultipleYieldSurface
- TsinghuaLiquefactionModel
- sanisand2004
- sanisand2008

# Material Models for Interface, Effective Stress

- Axial hard contact (hard surfaces)
- Axial soft contact (soil-concrete)
- Shear Elastic perfectly plastic shear contact
- Shear Elastic hardening plastic shear contact
- Shear Elastic hardening and softening shear contact



# Material Models for Interface, Normal and Shear

- BondedContact
- ForceBasedHardContact
- ForceBasedSoftContact
- ForceBasedCoupledHardContact
- ForceBasedCoupledSoftContact
- StressBasedHardContact\_ElPP1Shear
- StressBasedHardContact\_NonLinHardShear
- StressBasedHardContact\_NonLinHardSoftShear
- StressBasedSoftContact\_ElPP1Shear
- StressBasedSoftContact\_NonLinHardShear
- StressBasedSoftContact\_NonLinHardSoftShear
- StressBasedCoupledHardContact\_ElPP1Shear
- StressBasedCoupledHardContact\_NonLinHardShear
- StressBasedCoupledHardContact\_NonLinHardSoft
- StressBasedCoupledSoftContact\_ElPP1Shear
- StressBasedCoupledSoftContact\_NonLinHardShear
- StressBasedCoupledSoftContact\_NonLinHardSoftShear

# Material Models for Concrete

- DruckerPrager
- DruckerPragerArmstrongFrederickLE
- DruckerPragerArmstrongFrederickNE
- roundedMohrCoulomb
- FariaOliverCerveraConcrete
- uniaxial\_fiber\_concrete02

# Material Models for Steel

- vonMises
- vonMisesArmstrongFrederick
- uniaxial\_fiber\_steel01
- uniaxial\_fiber\_steel02

# Explicit and Implicit Constitutive Integration

- Explicit, forward Euler
  - Faster elastic-plastic calculations
  - Simplest elastic-plastic integration
  - Error accumulation
  - Tangent stiffness, used for explicit, noniterative global level
- Implicit, backward Euler
  - Slower elastic-plastic calculations
  - Most sophisticated elastic-plastic integration
  - Error controlled through user defined tolerance
  - Consistent stiffness, used for implicit, Newton iterations on global level



# FEM Equilibrium Iterations

- Global, finite element level equilibrium iterations
- Convergence criteria
  - Force, unbalanced, relative, abs. minimum check
  - Force, unbalanced, average
  - Force, unbalanced, absolute
  - Displacement, incremental, relative, abs. minimum check
  - Displacement, incremental, average
  - Displacement, incremental, absolute
  - Energy, incremental, relative, abs. minimum check
  - Energy, incremental, average
  - Energy, incremental, absolute

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**Modeling and Simulation**

**ESSI Modeling, Calibrations**

ESSI Simulation, Parameters

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# Sand Models

- Hyperbolic Drucker-Prager model with non-associated plastic flow and Armstrong-Frederick rotational kinematic hardening
- For example:
  - Density:  $\rho = 2000\text{kg/m}^3$
  - Wave velocity:  $V_s = 200\text{m/s}$   $V_p = 375\text{m/s}$
  - Strength parameter:  $c = 20\text{kPa}$ ,  $\phi = 36^\circ$
- From elasticity:
  - $E = 203.2\text{MPa}$   $\nu = 0.3$
- Target shear strength at 10m:  $\sigma = 10\text{m} * 2000\text{kg/m}^3 * 9.81\text{m/s}^2 = 196.2\text{kPa}$ 
  - $\tau = c + \sigma \tan(\phi) = 20\text{ kPa} + 196.2\text{kPa} * \tan(36^\circ) = 165\text{kPa}$

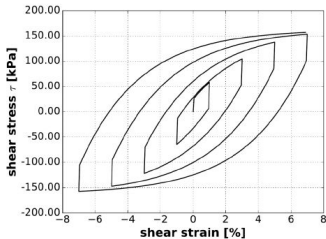
$$\nu = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)}$$

$$G = \rho V_s^2$$

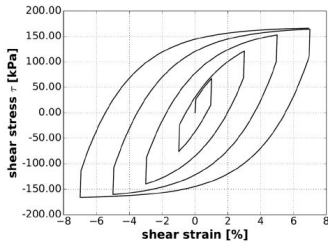
$$E = 2G(1 + \nu)$$



# Sand Models

**Initial trial:**

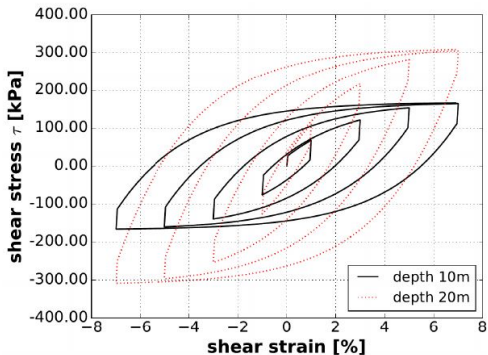
$$H_a = 3 \text{ MPa}$$
$$C_r = 25$$

**Fine tuned:**

$$H_a = 4 \text{ MPa}$$
$$C_r = 32$$

# Sand Models

Check for depth/pressure dependency



# Clay Models

- Density:  $\rho = 1954 \text{ kg/m}^3$
- Wave velocity:  $V_s = 200 \text{ m/s}$   $V_p = 375 \text{ m/s}$
- Undrained shear strength:  $S_u = 40 \text{ kPa}$

$$v = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)}$$

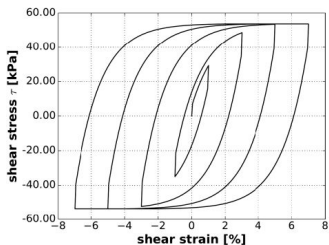
$$G = \rho V_s^2$$

$$E = 2G(1 + v)$$

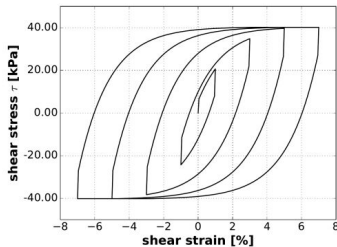
## Initial trial:

```
add material # <.> type vonMisesArmstrongFrederick
mass_density = 1954 kg/m3
elastic_modulus = 203.2 MPa
poisson_ratio = 0.30
von_mises_radius = 10 kPa
armstrong_frederick_ha = 5 MPa
armstrong_frederick_cr = 60
isotropic_hardening_rate = 0 kPa ;
```

# Clay Models

**Initial trial:**

$$H_a = 5 \text{ MPa}$$
$$C_r = 60$$

**Fine tuned:**

$$H_a = 3 \text{ MPa}$$
$$C_r = 50$$



# Interface/Contact/Joint Models

- Stress based soft contact with nonlinear hardening shear behavior
  - 9 parameters

```
add element # <.> type StressBasedSoftContact_NonLinHardShear
with nodes (<.>, <.>)
initial_axial_stiffness = <Pa>
stiffening_rate = <>
max_axial_stiffness = <Pa>
initial_shear_stiffness = <Pa>
axial_viscous_damping = <Pa*s>
shear_viscous_damping = <Pa*s>
residual_friction_coefficient = <.>
shear_zone_thickness = <m>
contact_plane_vector = (<.>, <.>, <.> );
surface_vector_relative_tolerance = <.>;
```

Normal penalty stiffness

Tangential initial stiffness

Damping in both directions

$\tan(\phi)$  where  $\phi$  is frictional angle

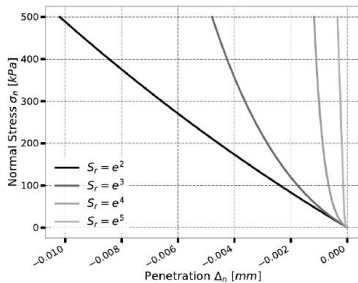
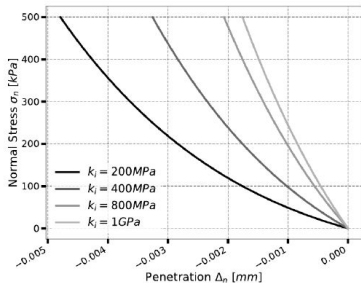
Contact shear zone thickness

Contact plane normal vector

Tolerance for automatic detection

# Interface/Contact/Joint Models

## ➤ Calibration of axial-normal direction

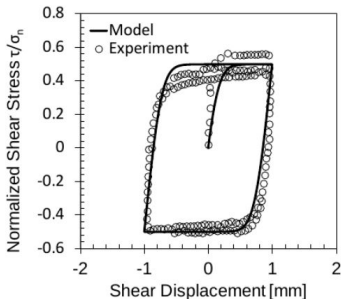


```
initial_axial_stiffness = 400 MPa  
stiffening_rate = 1000  
max_axial_stiffness = 20 GPa
```

# Interface/Contact/Joint Models

## ➤ Calibration of **shear** direction

### Smooth sand – Steel interface *Shahrour and Rezaie(1997)*



```
initial_shear_stiffness = 3 MPa  
residual_friction_coefficient = 0.50
```

# Interface/Contact/Joint Models

Mass concrete on soil	Friction coefficient ( $\tan \phi$ )	Friction angle ( $\phi$ )
Clean sound rock	0.70	35°
Clean gravel, gravel sand mixture, coarse sand	0.55 – 0.60	29° – 31°
Clean fine to medium sand, silty medium to coarse sand	0.45 – 0.55	24° – 29°
Fine sandy silt, nonplastic silt	0.35 – 0.45	19° – 24°
Very stiff clay	0.40 – 0.50	22° – 27°

Steel sheets against soil	Friction coefficient ( $\tan \phi$ )	Friction angle ( $\phi$ )
Clean gravel, sand-gravel mix, well graded rock fill	0.40	22°
Clean sand, silty sand-gravel mix, single size rock fill	0.30	17°
Fine sandy silt, nonplastic silt	0.20	11°

Formed concrete against soil	Friction coefficient ( $\tan \phi$ )	Friction angle ( $\phi$ )
Clean gravel, sand-gravel mix, well graded rock fill	0.40 – 0.50	22° – 27°
Clean sand, silty sand-gravel mix, single size rock fill	0.30 – 0.40	17° – 22°
Silty sand, gravel or sand mixed with silt and clay	0.30	17°
Fine sandy silt, nonplastic silt	0.25	14°

# Interface/Contact/Joint Models

## Other parameters with suggested values

- Axial and shear direction damping:

```
axial_viscous_damping = 50 Pa*s  
shear_viscous_damping = 50 Pa*s
```

- Shear zone thickness:

```
shear_zone_thickness = 0.01 m
```

- Contact normal vector: **from the 1<sup>st</sup> to 2<sup>nd</sup> contact node**, e.g., in z direction

```
contact_plane_vector = ( 0, 0, 1)
```

```
surface_vector_relative_tolerance = 1e-4
```

# Concrete Models

## ➤ Real-ESSI Domain Specific Language (DSL):

```
1 add material # <.> type uniaxial_concrete02
2 compressive_strength = <F/L^2>
3 strain_at_compressive_strength = <.>
4 crushing_strength = <F/L^2>
5 strain_at_crushing_strength = <.>
6 lambda = <.>
7 tensile_strength = <F/L^2>
8 tension_softening_stiffness = <F/L^2>;
```

Material parameters for monotonic compressive behavior, directly determined from physical tests.

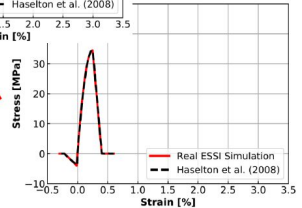
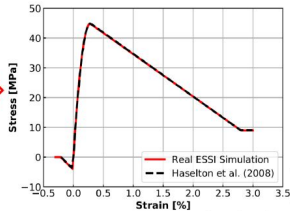
Material parameters for monotonic tensile behavior, directly determined from physical tests.

- Parameter **lambda** controls the unloading/reloading behavior of the concrete material model. If cyclic response of the model is important, physical tests involving cyclic loading is recommended to calibrate the value of lambda.

# Concrete Models

## ➤ Example

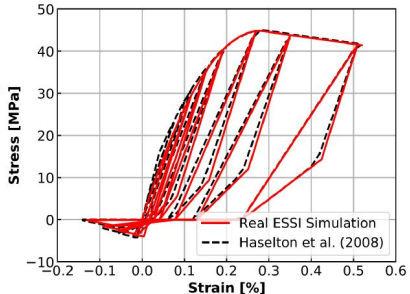
```
//CONFINED CONCRETE//  
add material # 1 type uniaxial_concrete02  
compressive_strength = -44.82e+6*Pa  
strain_at_compressive_strength = -0.0028  
crushing_strength = -8.96e+6*Pa  
strain_at_crushing_strength = -0.028  
lambda = 0.08  
tensile_strength = 4.0e+6*Pa  
tension_softening_stiffness = 2.0e+9*Pa;  
//UNCONFINED CONCRETE//  
add material # 2 type uniaxial_concrete02  
compressive_strength = -34.47e+6*Pa  
strain_at_compressive_strength = -0.0025  
crushing_strength = 0.0*Pa  
strain_at_crushing_strength = -0.004  
lambda = 0.08  
tensile_strength = 4.0e+6*Pa  
tension_softening_stiffness = 2.0e+9*Pa;  
//STEEL//  
add material # 3 type uniaxial_steel02  
yield_strength = 500e+6*Pa  
elastic_modulus = 200e+9*Pa  
strain_hardening_ratio = 0.001  
R0 = 18.0  
cR1 = 0.925  
cR2 = 0.15  
a1 = 0. a2 = 55. a3 = 0. a4 = 55. ;
```



# Concrete Models

## ➤ Example

```
//CONFINED CONCRETE//  
add material # 1 type uniaxial_concrete02  
compressive_strength = -44.82e+6*Pa  
strain_at_compressive_strength = -0.0028  
crushing_strength = -8.96e+6*Pa  
strain_at_crushing_strength = -0.028  
lambda = 0.08  
tensile_strength = 4.0e+6*Pa  
tension_softening_stiffness = 2.0e+9*Pa;  
  
//UNCONFINED CONCRETE//  
add material # 2 type uniaxial_concrete02  
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crushing_strength = 0.0*Pa  
strain_at_crushing_strength = -0.004  
lambda = 0.08  
tensile_strength = 4.0e+6*Pa  
tension_softening_stiffness = 2.0e+9*Pa;  
  
//STEEL//  
add material # 3 type uniaxial_steel02  
yield_strength = 500e+6*Pa  
elastic_modulus = 200e+9*Pa  
strain_hardening_ratio = 0.001  
R0 = 16.0  
cR1 = 0.925  
cR2 = 0.15  
a1 = 0. a2 = 55. a3 = 0. a4 = 55. ;
```





# Steel Models

## ➤ Real-ESSI Domain Specific Language (DSL):

```
1 add material # <.> type uniaxial_steel02
2   yield_strength = <F/L^2>
3   elastic_modulus = <F/L^2>
4   strain_hardening_ratio = <.>
5   R0 = <.>
6   cR1 = <.>
7   cR2 = <.>
8   a1 = <.>
9   a2 = <.>
10  a3 = <.>
11  a4 = <.> ;
```

Material parameters directly determined from physical experiments.

Material parameters that controls the shape of loading/unloading/reloading paths. Needs to be calibrated, default values usually work well.

## ➤ Recommended values:

- strain\_hardening\_ratio = 0.01, R0 = 18, cR1 = 0.925, cR2 = 0.15
- a1 = 0, a2 = 55, a3 = 0, a4 = 55

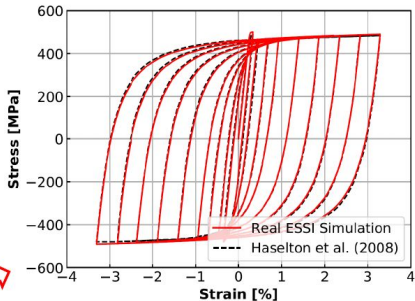
# Steel Models

## ➤ Example

```
//CONFINED CONCRETE//
add material # 1 type uniaxial_concrete02
compressive_strength = -44.82e+6*Pa
strain_at_compressive_strength = -0.0028
crushing_strength = -8.96e+6*Pa
strain_at_crushing_strength = -0.028
lambda = 0.08
tensile_strength = 4.0e+6*Pa
tension_softening_stiffness = 2.0e+9*Pa;

//UNCONFINED CONCRETE//
add material # 2 type uniaxial_concrete02
compressive_strength = -34.47e+6*Pa
strain_at_compressive_strength = -0.0025
crushing_strength = 0.0*Pa
strain_at_crushing_strength = -0.004
lambda = 0.08
tensile_strength = 4.0e+6*Pa
tension_softening_stiffness = 2.0e+9*Pa;

//STEEL//
add material # 3 type uniaxial_steel02
yield_strength = 500e+6*Pa
elastic_modulus = 200e+9*Pa
strain_hardening_ratio = 0.001
R0 = 18.0
cR1 = 0.925
cR2 = 0.15
a1 = 0. a2 = 55. a3 = 0. a4 =55. ;
```



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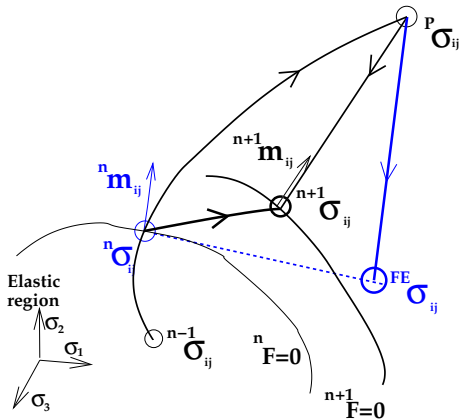
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ESSI Modeling, Calibrations  
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# Explicit and Implicit Constitutive Integration Algorithms



# Constitutive Integration, Explicit

```
1  define NDMaterial constitutive integration algorithm ↔  
    Forward_Euler;
```

```
1  define NDMaterial constitutive integration algorithm ↔  
    Forward_Euler_Subincrement  
2  number_of_subincrements = 10;
```

# Constitutive Integration, Implicit

```
1 define NDMaterial constitutive integration algorithm ↔  
    Backward_Euler  
2 yield_function_relative_tolerance = 1E-4  
3 stress_relative_tolerance = 1E-4  
4 maximum_iterations = 30;
```

```
1 define NDMaterial constitutive integration algorithm ↔  
    Backward_Euler_Subincrement  
2 yield_function_relative_tolerance = 1E-4  
3 stress_relative_tolerance = 1E-4  
4 maximum_iterations = 30  
5 allowed_subincrement_strain = 1E-5;
```



# Static Solution Advancement

```
1 define algorithm With_no_convergence_check;
```

```
1 define solver UMFPack;  
2 define convergence test Absolute_Norm_Displacement_Increment  
3     tolerance = 1E-6 maximum_iterations = 30 ;  
4 define algorithm Newton;  
5 define load factor increment 1/10;  
6 simulate 10 steps using static algorithm;
```

```
1 define convergence test Relative_Norm_Unbalanced_Force  
2     tolerance = 1e-2 minimum_absolute_tolerance = 1e4  
3     maximum_iterations = 50;  
4 define algorithm Newton;  
5 define solver UMFPack;  
6 define load factor increment 1/100;  
7 simulate 100 steps using static algorithm;
```



# Static Solution Advancement

```
1 // Gravity Stage -- Structure
2 new loading stage "Gravity_Structure";
3 include "Gravity_Load_Structure.fei";
4 include "Gravity_Load_Soil.fei";
5
6 if(IS_PARALLEL==0)
7   {define solver UMFPack;}
8 else
9   {define solver parallel petsc "-pc_type lu ↵
    -pc_factor_mat_solver_package mumps";}
10 define NDMaterial constitutive integration algorithm
11   Forward_Euler_Subincrement number_of_subincrements = 10;
12 define convergence test Relative_Norm_Unbalanced_Force
13   tolerance = 1e-2 minimum_absolute_tolerance = 1e4
14   maximum_iterations = 50;
15 define algorithm NewtonLineSearch;
16 gam = 0.7; bet = 0.25*(0.5+gam)^2;
17 define dynamic integrator Newmark with gamma = gam beta = bet;
18 simulate 1000 steps using transient algorithm time_step = 10*s;
```

# Dynamic Solution Advancement

```
1 dt=0.005*s;  
2 gamma_val=0.6;  
3 beta_val = 0.25*(0.5 + gamma_val)^2;  
4 define dynamic integrator Newmark with  
5   gamma = gamma_val beta = beta_val ;  
6 define algorithm With_no_convergence_check;  
7 define solver ProfileSPD;  
8 Nsteps = ceil(100*s/dt);  
9 simulate (Nsteps) steps using transient algorithm time_step = dt;
```

```
1 dt=0.01*s;  
2 alpha_val=-0.05;  
3 define dynamic integrator Hilber_Hughes_Taylor with  
4   alpha = alpha_val;  
5 define algorithm With_no_convergence_check;  
6 define solver ProfileSPD;  
7 Nsteps = ceil(100*s/dt);  
8 simulate (Nsteps) steps using transient algorithm time_step = dt;
```

# Dynamic Solution Advancement

```
1 //Dynamic: EQ
2 new loading stage "Dynamic";
3 motion_scale = 1.0;
4 include "Dynamic_Load.fei";
5 if(IS_PARALLEL==0)
6     {define solver UMFPack;}
7 else
8     {define solver parallel petsc "-pc_type lu ↔
9         -pc_factor_mat_solver_package mumps";}
9 define NDMaterial constitutive integration algorithm
10     Forward_Euler_Subincrement number_of_subincrements = 10;
11 define convergence test Relative_Norm_Unbalanced_Force
12     tolerance = 1e-2 minimum_absolute_tolerance = 1e3
13     maximum_iterations = 50;
14 define algorithm NewtonLineSearch;
15 gam = 0.7;
16 bet = 0.25*(0.5+gam)^2;
17 define dynamic integrator Newmark with gamma = gam beta = bet;
18 output every 10 steps;
19 simulate 25000 steps using transient algorithm time_step = 0.001*s;
```

# Outline

Introduction  
Motivation

Inelastic ESSI Analysis  
Analysis Phases  
Modeling and Simulation Components

Modeling and Simulation  
ESSI Modeling, Calibrations  
ESSI Simulation, Parameters

Summary

# Summary

- Expert engineer, analyst
- Full control of modeling, physical parameters
- Full control of simulation, numerical parameters
- Sensitivity of analysis results to modeling parameters
- Sensitivity of analysis results to simulation parameters
- Numerical modeling to predict and inform
- Education and Training is the key!