

## FEM AND STRESS CATEGORIES GROUPS IN THE CZECH STANDARD NTD A.S.I. SECTION III

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### ABSTRACT

The paper presents the Czech Standard NTD ASI Section III procedures, used for determining the stress categories groups  $(\sigma)_1$ ,  $(\sigma)_2$ ,  $(\sigma)_R$  and  $(\sigma_{aF})$ , when the finite element method is used.

### INTRODUCTION

ASME Code introduced the concept stress categories at the time, when strains and stresses were calculated using analytical relationships. It was assumed that each stress category will be gradually calculated separately and sequentially counted to groups. The software based on the finite element method (FEM) allows to calculate the total strains and stresses for given loads throughout the volume of the analysed components. In accordance with the original methodology should be calculated stresses the first divided into categories and then the selected categories would be adding to four "stress categories groups". The term "stress categories groups" was not defined in the ASME Code. For about 20 years later was this term defined in the Russian standard and in the Czech Standard NTD ASI Section III for assessment of the NPPs components of the WWER type (2018).

### FEM AND STRESS CATEGORIES GROUPS

The following four groups of stress categories are assessed for strength and durability (2018):  $(\sigma)_1$ ,  $(\sigma)_2$ ,  $(\sigma)_R$  and  $(\sigma_{aF})$ . Each of the four groups of stress categories is represented by a stress intensity calculated from the sum of the identical stress tensor components of the added stress categories. The stress categories groups reach the maximum on the surfaces of the wall.

The group  $(\sigma)_1$  is calculated from general membrane stresses  $\sigma_m$  derived by loads of mechanical origin.

The group  $(\sigma)_2$  is calculated from general membrane stresses  $\sigma_m$  or local membrane stresses  $\sigma_{mL}$  and general bending stresses  $\sigma_b$  from loads of mechanical origin.

The group  $(\sigma)_R$  represents the nominal stress intensity, calculated from the nonlinear stress distribution in the assessed cross section, derived from all loads acting at a given time. It is considered that the material was unloaded at least once.

The group  $(\sigma_{aF})$  represents the stress intensity, calculated at the notch location from the nonlinear distributed stress in the assessed cross section, derived from all loads acting at a given time.

The first two stress categories groups  $(\sigma)_1$  and  $(\sigma)_2$  are determined from stresses calculated using FEM for loads of mechanical origin. Both stress categories groups  $(\sigma)_1$  and  $(\sigma)_2$  require knowledge of the nominal stresses through the assessed cross section of the component wall. The stress categories groups  $(\sigma)_R$  (range of stress) and  $(\sigma)_{aF}$  (fatigue) are determined from stresses calculated using FEM for all loads including heat inserted by flowing medium into the component wall. Groups categories group  $(\sigma)_R$  is determined from the nominal stresses calculated for the planned period of component operation. Stress categories group  $(\sigma)_{aF}$  represents the intensity of strains in the surface notch assessed cross section. Each of the stress categories groups can to be assessed only in the important cross section.

Definitions of terms of groups of stress categories have been used in the development of SW STATES in the Institute of Applied Mechanics Brno since 1986. SW STATES (2358/96) is used to assess the limit states of strength; it uses stresses calculated by FEM. It was used to assess the strength and durability of equipment I. and II. Temelin NPP circuits. The procedures inserted into the STATES software were used in the creation of the NTD AME Section III for assessment of the VVER NPPs components.

The calculated stresses are assessed in the cross section of a rotationally symmetric shell (Figure 1), or on the line in the general shape of the body, eg at the junction of the nozzle with the shell, Figure 2.

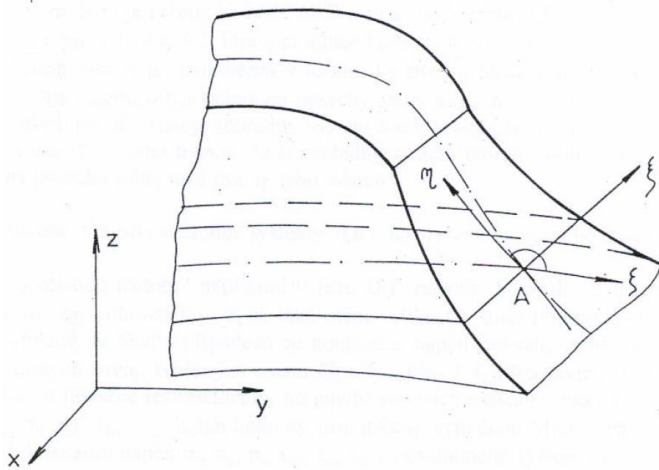


Figure 1. Assessed cross section of a rotationally symmetric shell.

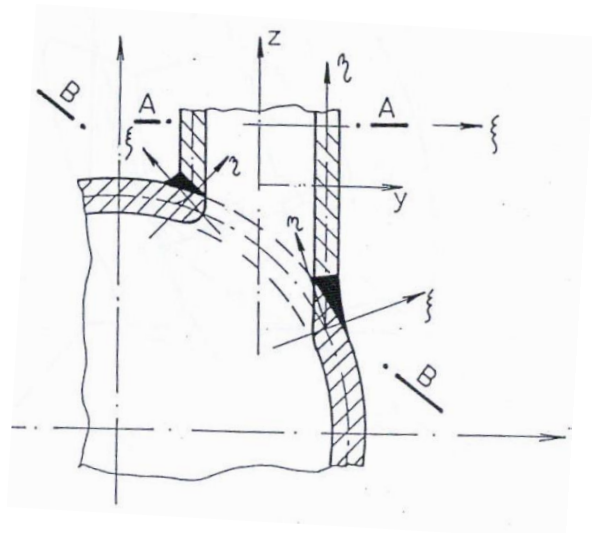


Figure 2. Assessed line in the junction of the nozzle with the shell.

### CALCULATION OF NOMINAL STRESSES

In all peaks of the elements lying on the axis  $\xi$  (Fig. 3), we know all the members of the stress tensor  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$ ,  $\tau_{xy}$ ,  $\tau_{yz}$  and  $\tau_{zx}$ . All these stresses in the cross section taken by the shell wall to have a non-linear distribution. However, the linear distribution of stresses in the cross section (or line) of them can be calculated only from those members of the stress tensor for which it is in accordance with their physical nature, eg  $\sigma_\zeta$ ,  $\sigma_\eta$ .

The linearization of the stress tensor components  $\sigma_\eta$  and  $\sigma_\zeta$  is based on the fact, that both nonlinear and linear distributions of these stress tensor components have the same resultant. Linearizing the other components of the stress tensor would be problematic, (2400/97).

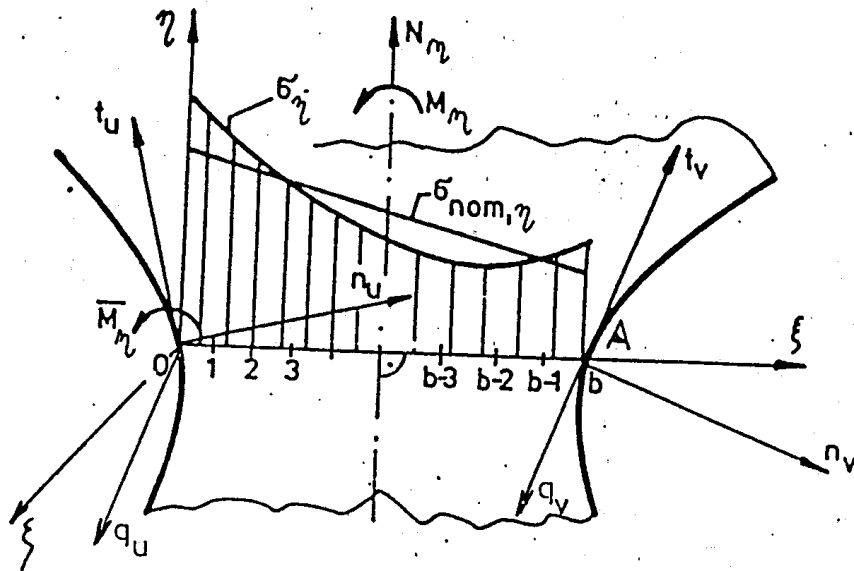


Figure 3. Stresses linearization. The nonlinear course of stresses can be expressed analytically, or the length of the section (or line) can be divided into short sections and in them take the mean value of stress, calculated from the values from the extreme points of the short section.

At surface points 0 and A we know that on a surface loaded with pressure  $p$ , the stress is in the direction perpendicular to the surface, then  $\sigma_n = -p$  and on an unloaded surface  $\sigma_n = 0$ . If the main stress is on the wall surface in the direction of axes  $n$ ,  $t$  and  $q$  then the components of the shear stress tensor  $\tau_{nt} = \tau_{nq} = \tau_{tq} = 0$ . In the coordinate system  $ntq$  we know 4 of the 6 components of the tensor of nominal stresses  $\sigma_n$ ,  $\tau_{nt}$ ,  $\tau_{nq}$  and  $\tau_{tq}$  and in the coordinate system  $\xi\eta\zeta$  two components of the nominal stress tensor  $\sigma_\eta$  and  $\sigma_\zeta$  a total of 6 components from two related nominal stress tensors.

The matrix notation (1) of the system of equations is used to calculate the unknown components of the stress tensor  $\sigma_\zeta$ ,  $\sigma_q$ ,  $\sigma_n$ ,  $\tau_{\zeta\xi}$ ,  $\tau_{\xi\eta}$ ,  $\tau_{\eta\zeta}$  from the known components of the stress tensor  $\sigma_n$ ,  $\sigma_\zeta$ ,  $\sigma_\eta$ ,  $\tau_{qn} = \tau_{nt} = \tau_{tq} = 0$ . The directional cosines of the axes of the coordinate system  $\zeta\xi\eta$  with respect to the axes of the coordinate system  $qnt$  are written symbolically in the matrix:  $\bar{\alpha}_i \cong \cos \bar{\alpha}_i$ ;  $\bar{\beta}_i \cong \cos \bar{\beta}_i$ ;  $\bar{\gamma}_i \cong \cos \bar{\gamma}_i$ , where  $i = \zeta, \xi, \eta$ .

$$\begin{pmatrix} \sigma_\xi \\ \sigma_q \\ \sigma_t \\ \tau_{\zeta\xi} \\ \tau_{\xi\eta} \\ \tau_{\eta\zeta} \end{pmatrix} = \begin{pmatrix} -1; & \bar{\alpha}_\xi^2; & \bar{\gamma}_\xi^2; & 0; & 0; & 0 \\ 0; & \bar{\alpha}_\zeta^2; & \bar{\gamma}_\eta^2; & 0; & 0; & 0 \\ 0; & \bar{\alpha}_\zeta^2; & \bar{\gamma}_\zeta^2; & 0; & 0; & 0 \\ 0; & \bar{\alpha}_\zeta \cdot \bar{\alpha}_\xi; & \bar{\gamma}_\zeta \cdot \bar{\gamma}_\xi; & -1; & 0; & 0 \\ 0; & \bar{\alpha}_\xi \cdot \bar{\alpha}_\eta; & \bar{\gamma}_\xi \cdot \bar{\gamma}_\eta; & 0; & -1; & 0 \\ 0; & \bar{\alpha}_\eta \cdot \bar{\alpha}_\zeta; & \bar{\gamma}_\eta \cdot \bar{\gamma}_\zeta; & 0; & 0; & -1 \end{pmatrix} \cdot \begin{pmatrix} -\bar{\beta}_\xi^2 \cdot \sigma_n \\ -\bar{\beta}_\zeta^2 \cdot \sigma_n + \sigma_\zeta \\ -\bar{\beta}_\eta^2 \cdot \sigma_n + \sigma_\eta \\ -\bar{\beta}_\zeta \cdot \bar{\beta}_\xi \cdot \sigma_n \\ -\bar{\beta}_\xi \cdot \bar{\beta}_\eta \cdot \sigma_n \\ -\bar{\beta}_\eta \cdot \bar{\beta}_\zeta \cdot \sigma_n \end{pmatrix} \quad (1)$$

## NOTCH STRAIN INTENSITY

The Neuber principle (Figure 4) is used to calculate the strain intensity, if the stress intensity is known, calculated by the FEM for the elastic stress state, (1975 and 2331/96). The material with nonlinear reinforcement above the limit of proportionality  $f_e$  is assumed.

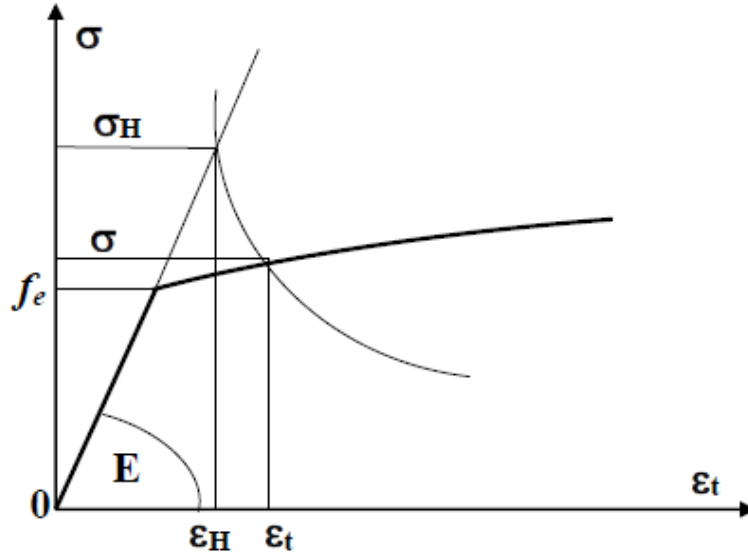


Figure 4. Material with nonlinear reinforcement above the limit of proportionality  $f_e$ . Neuber principle.

The strain intensity  $\varepsilon_t$  and the stress intensity  $\sigma_H$  with a monotonic load increase above the proportionality limit  $f_e$  are calculated from the relations:

$$\varepsilon_{t,i} = \frac{f_e}{E} \left[ \frac{\sigma_{H,i}}{f_e} \right]^{2/(m+1)} ; \quad \sigma = f_e \left[ \frac{E \cdot \varepsilon_{t,i}}{f_e} \right]^m \quad (2)$$

At stress below the proportionality limit  $f_e$  is:

$$\sigma_i = \sigma_{H,i}; \quad \varepsilon_{t,i} = \varepsilon_{H,i} = \sigma_{H,i}/E \quad (3)$$

The strain range  $\Delta\varepsilon_{t,i}$  and the stress range  $\Delta\sigma_i$  when the load on the hysteresis loop branch exceeds the value  $2f_e$  are calculated from the relations:

$$\Delta\varepsilon_{t,i} = \frac{2f_e}{E} \left[ \frac{\Delta\sigma_{H,i}}{2f_e} \right]^{2/(m+1)} ; \quad \Delta\sigma_i = 2f_e \left[ \frac{E \cdot \Delta\varepsilon_{t,i}}{2f_e} \right]^m \quad (4)$$

At stress below the two proportionality limits  $2f_e$  is:

$$\Delta\sigma_i = \Delta\sigma_{H,i}; \quad \Delta\varepsilon_{t,i} = \Delta\varepsilon_{H,i} = \Delta\sigma_{H,i}/E \quad (5)$$

The exponent  $m$  and the proportionality limit  $f_e$  are calculated from the relations:

$$m = \frac{0.73 \cdot \lg \left[ \frac{(1+0.014 \cdot Z) f_u}{f_y} \right]}{\lg \left[ \frac{2.3E \cdot \lg [100/(100-Z)]}{0.002E + f_y} \right]} ; \quad f_e = \left[ \frac{f_y}{(0.002E + f_y)^m} \right]^{\frac{1}{1-m}} \quad (6)$$

Where  $f_u$  - tensile strength;  $f_y$  - yield strength;  $E$  - Young modulus at temperature  $T$ .

## CHANGE OF PRINCIPAL STRESSES DIRECTIONS

When the direction of principal stresses changes over time, the directional cosines of principal stresses  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ , which give the greatest stress intensity  $\sigma_i$  for the whole assessed period. These stresses are found at the surface points on the assessed cross section (line). The directions of these principal stresses are the directions of the new  $XYZ$  coordinate system. For this most adverse stress state, the principle stress  $\sigma_1$  indicates the  $X$  direction,  $\sigma_2$  the  $Y$  direction and  $\sigma_3$  the  $Z$  direction. Subsequently, it is necessary to calculate the directional cosines in the  $XYZ$  coordinate system of all principal stresses from other time points.

The principal stress which is closest to the  $X$  axis to give index 1, which is closest to the  $Y$  axis to give index 2 and which is closest to the  $Z$  axis to give index 3. The criterion for assigning the principle stresses to one of the  $X$ ,  $Y$  and  $Z$  axes is the value of the cosine lying in the interval  $(\sqrt{3}/3; 1)$ . The directional cosines of the principal stress  $\sigma_j$ , where  $j = 1, 2$  and  $3$  in the new coordinate system  $XYZ$  are calculated from the relations:

$$\begin{aligned} \cos \alpha_{mj} &= \cos(2 * \arccos \varphi_1); & \cos \beta_{mj} &= \cos(2 * \arccos \varphi_2) \\ \cos \gamma_{mj} &= \cos(2 * \arccos \varphi_3), \end{aligned} \quad (7)$$

where:

$$\begin{aligned} \varphi_1 &= \sqrt{1 - 0,25 \cdot [(\cos \alpha_i - \cos \alpha_{1m})^2 + (\cos \beta_i - \cos \beta_{1m})^2 + (\cos \gamma_i - \cos \gamma_{1m})^2]} \\ \varphi_2 &= \sqrt{1 - 0,25 \cdot [(\cos \alpha_i - \cos \alpha_{2m})^2 + (\cos \beta_i - \cos \beta_{2m})^2 + (\cos \gamma_i - \cos \gamma_{2m})^2]} \\ \varphi_3 &= \sqrt{1 - 0,25 \cdot [(\cos \alpha_i - \cos \alpha_{3m})^2 + (\cos \beta_i - \cos \beta_{3m})^2 + (\cos \gamma_i - \cos \gamma_{3m})^2]}, \end{aligned} \quad (8)$$

where  $\cos \alpha_i$ ,  $\cos \beta_i$  and  $\cos \gamma_i$  are the directional cosines of the principal stress  $\sigma_i$  in the original  $xyz$  coordinate system. In the original global  $xyz$  coordinate system, the new  $X$  axis has directional cosines

marked  $\cos \alpha_{1m}$ ,  $\cos \beta_{1m}$  and  $\cos \gamma_{1m}$ , the Y axis has directional cosines marked  $\cos \alpha_{2m}$ ,  $\cos \beta_{2m}$  and  $\cos \gamma_{2m}$ , and the Z axis has directional cosines marked  $\cos \alpha_{3m}$ ,  $\cos \beta_{3m}$  and  $\cos \gamma_{3m}$ .

The directional cosines in the original global coordinate system  $xyz$  for the triaxle stress state  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$ ,  $\tau_{xy}$ ,  $\tau_{yz}$  and  $\tau_{zx}$  are calculated from (9).

$$\cos \alpha = \pm \sqrt{\frac{1}{1 + \left(a_1 + \frac{b_1 * b_2}{a_2}\right)^2 + \left(\frac{b_2}{a_2}\right)^2}} ; \quad \cos \gamma = \frac{b_2}{a_2} * \cos \alpha ;$$

$$\cos \beta = a_1 * \cos \alpha + b_1 * \cos \gamma \quad (9)$$

Where:

$$a_1 = -\frac{\tau_{xy}}{\sigma_y - \sigma}; \quad b_1 = -\frac{\tau_{yz}}{\sigma_y - \sigma};$$

$$a_2 = 1 - \frac{\tau_{yz}^2}{(\sigma_z - \sigma) * (\sigma_y - \sigma)}; \quad b_2 = -\frac{1}{\sigma_z - \sigma} * \left( \tau_{zx} - \frac{\tau_{yz} * \tau_{xy}}{\sigma_y - \sigma} \right) \quad (10)$$

When calculating the directional cosines of the principle stress  $\sigma_1$ , we substitute  $\sigma_1$  for  $\sigma$  into (10); when calculating the directional cosines of the principle stress  $\sigma_2$ , we substitute  $\sigma_2$  for  $\sigma$  and when calculating the directional cosines of the principle stress  $\sigma_3$ , we substitute  $\sigma_3$  for  $\sigma$ .

## CONCLUSIONS

The paper presents any of the procedures used for determining the stress categories groups  $(\sigma)_1$ ,  $(\sigma)_2$ ,  $(\sigma)_R$  and  $(\sigma_{aF})$  when the FEM is used.

The calculation of stress category groups is an executive part of SW STATES, used for assessment of limit states.

SW STATES made it possible to use complex relationships to assess the strength and service life of NPP equipment and saved time and made it possible to assess several structural designs of significantly stressed parts of structures.

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