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MODEL UNCERTAINTY IN DESIGN AND RELIABILITY ASSESSMENTS OF STRUCTURES SUBJECTED TO IMPACT

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ABSTRACT

In Structural Engineering, uncertainties may be classified in the following groups: (1) Uncertainty derived from the random nature of loads and external actions, (2) Uncertainties concerning material properties and dimensions. Both (1) and (2) result from the inherent variability observed in most actions on structures (wind, earthquakes, temperature) and in the geometrical and mechanical properties of structural members. (3) Model Uncertainty. A basic problem of Structural Engineering consists of the determination of the response of a structural system to a given excitation. For such purpose different models of mechanical or physical behaviour are resorted to, which introduce model uncertainty in the assessment. (4) Phenomenological Uncertainty. (5) Human Error. While in most engineering problems the influence of the last three groups of uncertainties is marginal, in case of impact loading model uncertainty plays a dominant role and hence it cannot be ignored. In spite of its importance, there are no widely accepted criteria to account for model uncertainty in such situations. In the present paper different approaches are considered: use of proposed formulas to predict features of the structural response, such as penetration or perforation, numerical response analysis of the coupled projectile-structural system, numerical analysis of the structure subjected to assumed loads induced by the projectile. Several round-robin experiments to assess model uncertainty in structural impact and related problems are also reviewed and, on that basis, suggestions are presented to account for model uncertainty in the adoption of design or reliability decisions.

INTRODUCTION

In Structural Engineering, as discussed for instance by Melchers (1987), uncertainties may be classified in the following groups: *(1) Uncertainty derived from the random nature of loads and external actions, (2) Uncertainties concerning material properties and dimensions*. Both (1) and (2) result from the inherent variability observed in most actions on structures (wind, earthquakes, temperature) and in the geometrical and mechanical properties of structural members and usually constitute the *only uncertainties explicitly considered* in Codes or Regulations or in risk assessments. *(3) Model Uncertainty*. A basic problem of Structural Engineering consists of the determination of the response of a structural system to a given excitation. For such purpose different *models of mechanical or physical behaviour*, as well as a variety of numerical approximations are resorted to, which introduce *model uncertainty* in our assessments. The latter should not be confused with statistical model uncertainty, term that refers to the choice of probability distribution functions for the relevant variables. *(4) Phenomenological Uncertainty*. Every reliability analysis or design procedure implicitly accepts the assumption that all possible failure modes were duly taken into consideration. However, failure may also occur due to a phenomenon unknown at the time when the system was designed. Uncertainty about the existence of a relevant failure mode is known as *Phenomenological Uncertainty*. *(5) Human Error*. In any engineering project there is also a non-zero chance of a design or construction error that may lead to failure. The problem was discussed by Melchers

(1994), but its consideration in structural design or in reliability assessments still presents difficulties. While in most engineering problems the influence of the last three groups of uncertainties is marginal and may often be neglected, in case of impact loading model uncertainty plays a dominant role (Riera, 2012, 2013). In spite of its importance, there are no widely accepted criteria to account for model uncertainty in such situations.

In the present paper different approaches are examined: the use of proposed formulas to predict features of the structural response, such as penetration or perforation (Kosteski *et al*., 2014, 2015), detailed numerical response analysis of the coupled projectile-structural system and finally numerical analysis of the structure subjected to assumed loads induced by the projectile. Several round-robin experiments to assess model uncertainty in structural impact and related problems are also reviewed and, on that basis, suggestions are presented to account for model uncertainty in the adoption of design or reliability decisions.

RESPONSE CHARACTERIZATION IN PROJECTILE IMPACT

As thoroughly described by Kosteski *et al*. (2014), the determination of the response of structures subjected to impact constitutes one of the most difficult problems of non-linear structural dynamics. In practical engineering design, the basic features that characterize the response of quasi-fragile materials such as concrete or rock are schematically shown in Figure 1, which shows a possible damage distribution in a large solid, represented by a 3D half-space, subjected to normal impact of a projectile, modelled in turn as a cylinder of diameter d and length l. When damage occurs, an indentation x remains in the impacted body, herein designated *penetration*. A crater may also develop around the impact point with the expulsion of debris in the direction opposite to the impact direction, which is designated *spalling*. When the target structure is a wall or shell, the problem is more complex, since in addition to the previously mentioned phenomena of penetration and spalling, *scabbing* and *perforation* may also occur, as illustrated by Fig. 1(b). Scabbing occurs when a compression pulse induced upon impact is reflected at the opposite plate surface, becoming a tension pulse with the same intensity. As this tension pulse travels backward, cracking along a plane parallel to the plate middle surface may occur, thus beginning the formation of a crater, in materials characterized by low tensile strength.

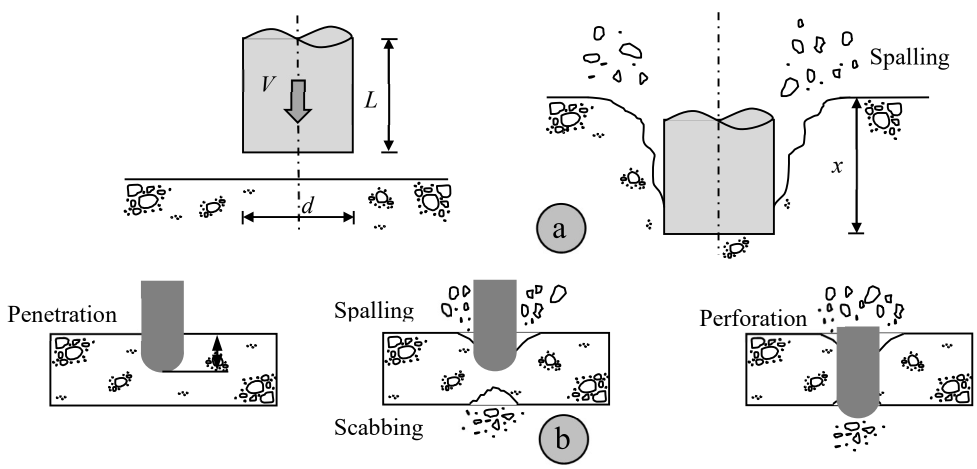

Figure 1. (a) Normal impact of a cylindrical projectile against the surface of a half-space of quasi-fragile material showing both penetration and spalling and (b) idem, against a thin wall (Kosteski *et al*., 2015).

Perforation implies, when the residual velocity of the projectile is different from zero, complete failure of the wall or shell. A limit situation corresponds to the case in which this residual velocity is zero, *i.e.* the projectile perforates the structure but does not pass through. Table 2.1 presents a summary of the terms used to define the impact problem, as well as the notation employed in the following sections. Note that the perforation thickness denotes the minimum thickness required to prevent perforation, for a given set of remaining parameters. Similarly, the scabbing thickness is the minimum plate or shell thickness to avoid scabbing, all other parameters being fixed. The practical relevance of the prediction equations discussed herein is evident: it is often sufficient to determine the penetration depth to define the depth at which a duct should be buried or embedded within a concrete wall to avoid duct failure in case of impact, or determine the thickness of a protection structure to assure that scabbing cannot occur. Their usefulness is nevertheless not restricted to routine engineering applications: whenever elaborate analyses are conducted to assess the non-linear dynamic response of structures subjected to projectile impact, it is advisable to resort, if at all possible, to gross checks of the numerical results by comparison with the prediction equations.

Table 1. Designation and notation of relevant quantities in standard impact problem.

	Description	Notation	Unit
Response parameter	Penetration depth	x_p	*m*
	Perforation thickness	h_p	*m*
	Scabbing thickness	h_s	*m*
Projectile parameters	Mass	m	*kg*
	Diameter of projectile	d	*m*
	Impact Velocity	V	*m/sec*
	Nose shape factor	N	--
Target parameters	Target plate or shell thickness	h	*m*
	Concrete compressive strength	f_c'	*Pa*
	Concrete tensile strength	f_t	*Pa*

FORMULAS TO PREDICT PENETRATION OF CONCRETE STRUCTURES

With the aim of predicting the penetration depth, shown in Figure 1(a), several empirical and semi-empirical formulas were proposed in the technical literature. Adeli and Amin (1985) as well as Rahman *et al*. (2010), present reviews and comparisons between several equations. Li (2012) presents an extensive and detailed review of engineering formulas proposed to predict penetration and other impact effects in concrete. The most widely used of these formulas were examined later by Kosteski *et al*. (2015) in the comparative study briefly described below.

Modified Petry formula: The oldest penetration prediction equation, dating back to 1910, is the Petry formula. In the original formula the so-called concrete penetrability factor *k* was considered independent of the strength of concrete and of reinforcement. The modified Petry formula (1), in which the concrete penetrability factor *k* was modified to account for different degrees of reinforcement, in SI units, is:

$$x_p = k\frac{m}{d^2}log_{10}\left(1+\frac{V^2}{19974}\right) \quad (1)$$

The parameters are defined in Table 1. The additional parameter *k*, that takes into account the influence of reinforcement, has a value $k = 6.36 \times 10^{-4}$ for unreinforced concrete, $k = 3.39 \times 10^{-4}$ for standard reinforced concrete and $k = 2.26 \times 10^{-4}$ for specially reinforced concrete. Later, the coefficient *k* was modified by Amirikian (1950), who considers that *k* is also a function of f_c'.

Modified National Defense Research Committee (NDRC) formula: The Ordnance Department of US Army and Ballistic Research Laboratory (BRL) carried before mid-20th Century an experimental study on the local effects of hard projectiles impact on concrete structures, which led to the theory of penetration of rigid missiles into massive concrete target suggested in 1946 by the *NDRC*. This theory of penetration enabled not only to calculate the total penetration depth, but also to determine the impact force and penetration depth versus time histories. The *NDRC* approach proposed to determine the penetration depth from a G – function equations. Later, based on both theoretical and experimental data, Kennedy (1966) suggested a modification of this function accounting for the compressive strength or unconfined compressive strength of concrete. Then the modified *NDRC* formula, in SI units, is then expressed as:

$$G = \frac{3.8\times10^{-5} N m}{d\sqrt{f_c'}}\left(\frac{V}{d}\right)^{1.8} \tag{2}$$

In which *N* is a *nose shape factor* for the projectile, established as 0.72 for flat nose, 1.00 for average bullet nose (spherical end), 0.84 for blunt nosed bodies, and 1.14 for very sharp nose. *G* is given by the following functions of x_p/d :

$$\begin{aligned} \frac{x_p}{d} &= 2G^{0.5} \qquad \text{for } \frac{x_p}{d} \le 2 \\ \frac{x_p}{d} &= G + 1 \qquad \text{for } \frac{x_p}{d} > 2 \end{aligned} \tag{3}$$

Haldar and Miller formula: Haldar and Miller (1982) and Haldar and Hemieh (1984) introduced a dimensionless *impact factor I* defined as:

$$I = \frac{NmV^2}{d^3 f_c} \tag{4}$$

In which *N* is the nose shape factor defined in the modified *NDRC* formula (eq.2). Haldar *et al*. (1983) assumed that there is a functional relationship between the penetration to diameter ratio and the impact factor. Three linear equations for the x_p/d ratio were fitted to experimental data, each valid for a different range of values of *I,* as indicated below:

$$\begin{aligned} \frac{x_p}{d} &= 0.2551I + 0.0308 \qquad \text{for } 0.3 \le I \le 4.0 \\ \frac{x_p}{d} &= 0.0567I + 0.6740 \qquad \text{for } 4.0 \le I \le 21 \\ \frac{x_p}{d} &= 0.0299I + 1.1875 \qquad \text{for } 21 \le I \le 455 \end{aligned} \tag{5}$$

Hughes formula: Hughes (1984) assumed that the resistance to penetration of rigid projectiles is an increasing linear function, followed by a decreasing parabolic function, of the distance to the target surface, proposing Eq. (6) for the penetration depth. Hughes (1984) defined an impact factor I' [the term in parenthesis in the numerator of eq. (6)] depending on the concrete tensile strength f_t. rather than its compressive strength, which is used in most other formulations. Hughes (1984) who also accounts for the influence of strain rate on the tensile strength by introducing a Dynamic Increase Factor (DIF) determined by an empirical calibration with penetration observations.

$$\frac{x_p}{d} = \frac{0.19\ '\left(\frac{mV^2}{d^3 f_t}\right)}{\left[1+12.3 ln\left(1+0.03\frac{mV^2}{d^3 f_t}\right)\right]} \tag{6}$$

where K' is a nose shape factor specified as 1.00 for flat nosed missiles, 1.12 for blunt nosed missiles, 1.26 for average bullet nose (spherical end), and 1.39 for very sharp nose.

Riera formula: Riera (1989) considered the impact of a circular projectile of diameter *d* and mass *m* impinging normally with velocity *V* against the surface of a massive concrete structure. It was assumed that the projectile material is rigid-perfectly plastic and that the resistance to penetration of the target is a monotonically increasing function of depth x_p that tends asymptotically to a limiting value:

$$x_p = \frac{\beta_1 - \beta_2 exp\left(-cx_p/d\right)}{N} \frac{\pi d^2 f_c'}{4} \tag{7}$$

in which f_c' denotes the unconfined compressive strength of the material and *N* is the nose shape factor for the projectile, as in the modified NDRC formula, while β_1, β_2 and *c* are non-dimensional coefficients. The unknown coefficients in eq. (7) were then evaluated by means of a nonlinear regression analysis using the same data employed by Haldar and Miller (1982), to which five observations corresponding to full-scale tests for low values of *I* were added. The resulting equation is:

$$I = 30.97\left(x_p/d\right) - 51.29\left[1 - exp\left(-0.598\, x_p/d\right)\right] \tag{8}$$

Although the global goodness of fit of eq. (8) is satisfactory, Riera (1989) contends that the concrete tensile strength f_t should be more appropriate than the compressive strength to predict both penetration or perforation of concrete, as proposed earlier by Hughes (1984). Therefore, the impact factor was redefined by Riera (1989) as:

$$I = \frac{NmV^2}{2\pi d^3 f_t} \tag{9}$$

Riera (1989) notes that the impact factors for *normal strength concrete* calculated according to either eqs. (4) or (9) are approximately the same, while prediction equation (8) is not altered. However, when eq. (8) is applied to other target materials, the impact factor should be calculated by means of eq. (9). Moreover, Riera (1989) derived the only penetration equation known to the authors that accounts for the influence of the slenderness, measured by the length to diameter ratio *l/d*, for *soft* projectiles, *i.e.* for projectiles with negligible crushing strength. In such cases, the impact factor *I* is related to the penetration to diameter ratio x_d/d by the following equation:

$$I = \frac{\pi}{2}\left\{\beta_1\left(x_p/d\right) - \beta_2 \frac{1 - exp\left(-cx_p/d\right)}{c} + \frac{1}{2}\left[\beta_1 - \beta_2 exp\left(-c\, x_p/d\right)\right]\left(l/d\right)\right\} \tag{10}$$

BRIEF DESCRIPTION OF KOJIMA (1991) IMPACT TESTS

A series of reinforced concrete plates subjected to projectile impact were tested by Kojima (1991). The 1.2m square plates were 0.12m thick, and were fixed to the supports as shown in Figure 2(a). The plates were double-layer reinforced concrete slabs, with reinforcement ratios of 0.6% on one side, for both longitudinal and transversal bars. Figure 2(a) presents a view of the plate and distribution of the reinforcement. Shear reinforcing bars were not used. The definitions of the three different types of failure reported by Kojima (1991) also adopted in the present paper, as discussed in Section 2, are illustrated in Figure 1(b). The projectiles had a hemispherical steel nose, 2kg mass, 6cm diameter and 10 cm length as shown in Figure 2(b). The impact velocity was determined by the time of flight and distance between two points. Reactions were measured by load cells installed at the supports on the four corners of the slab. Kojima (1991) studied several cases with different impact velocities and slab characteristics, and the results were compared with empirical expressions. The nominal compressive strength of concrete was 27MPa and its tensile strength 2.2MPa. Three of these tests were simulated in the study, corresponding to impact velocities equal to 215m/s, 164m/s and 95m/s, respectively, identified as tests R-12-X, R-12-Y and R-12-Z.

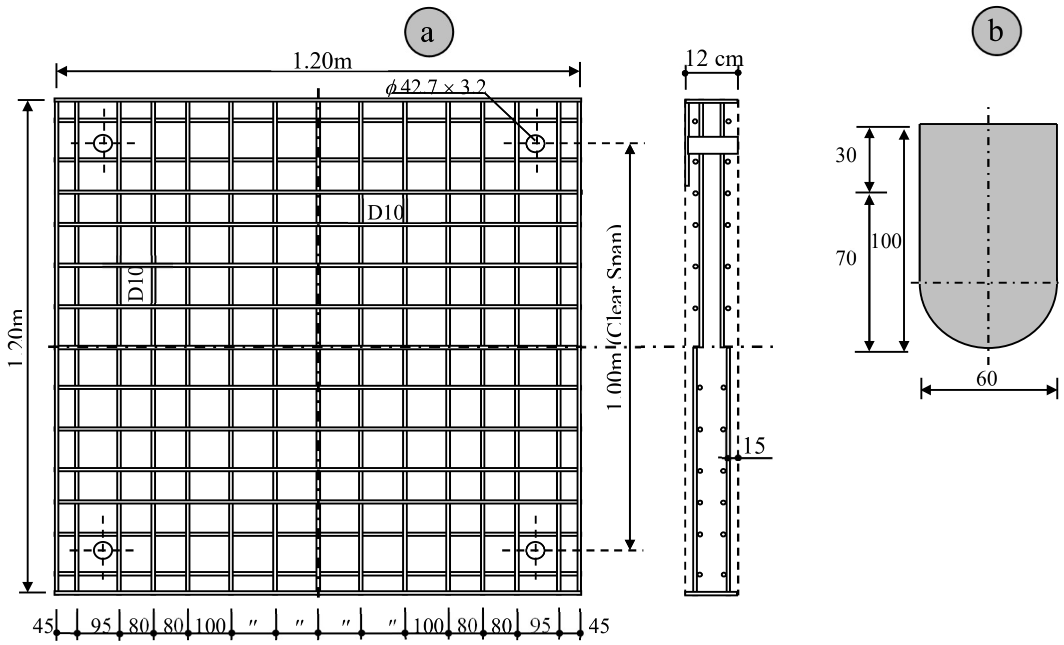

Figure 2. (a) Layout of the plate here studied, (D10 indicates the rebar diameter of 10mm), (b) Hard-nosed projectile (Kojima 1991).

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ESTIMATION OF MODEL ERROR OF EMPIRICAL FORMULAS

Comparisons between empirical formulae for the penetration depth (x_p) and experimental results for projectiles impacting against a concrete plate tested by Kojima (1991), with initial velocities of 215 m/s, 164 m/s e 95 m/s, are presented in Table 2 (Kosteski *et al*., 2015). The various empirical equations assessed in the study predict similar penetration depths, with an average coefficient of variation around 0.08 in the range of velocities considered.

Table 2. Comparisons between empirical formulae for penetration with experimental results for projectile impacts with different velocities (Kosteski *et al*., 2015).

Penetration Depth x_p (mm)	Projectile impact velocity		
	V= 95 m/s	V= 164 m/s	V= 215 m/s
Modified Petry	30.5	69.8	98.0
NDRC	44.9	73.4	93.7
Haldar and Miller	49.3	71.8	94.4
Hughes	52.5	84.3	112.9
Riera	51.5	95.8	132.3
Mean value of empirical formulae	**45.7**	**79.0**	**106.3**
Standard deviation of formulae	4.50	5.47	8.25
Coefficient of variation	0.10	0.07	0.08
Experimental	**44**	**Not available**	**Perforation**

Since penetration equations intend to assess the response of a semi-space, in case of plate or shell structures their predictions should be regarded with care when the penetration to thickness ratio exceeds about 0.4 and are not applicable when this ratio approaches unity. Thus, only the penetration formulas predictions in the first column of Table 2 (V= 95 m/s) may be compared with the experimental value. It may be then concluded that the coefficient of variation of 10% determined for penetration formulas *is a lower bound* both for perforation formulas and for numerical predictions of structural response in impact problems. DEM analysis performed by Kosteski *et al.* (2014), shown in Fig. 3, are compatible with the previous 10% estimate, but do not allow a statistical validation, for which purpose predictions with *other models* would be required.

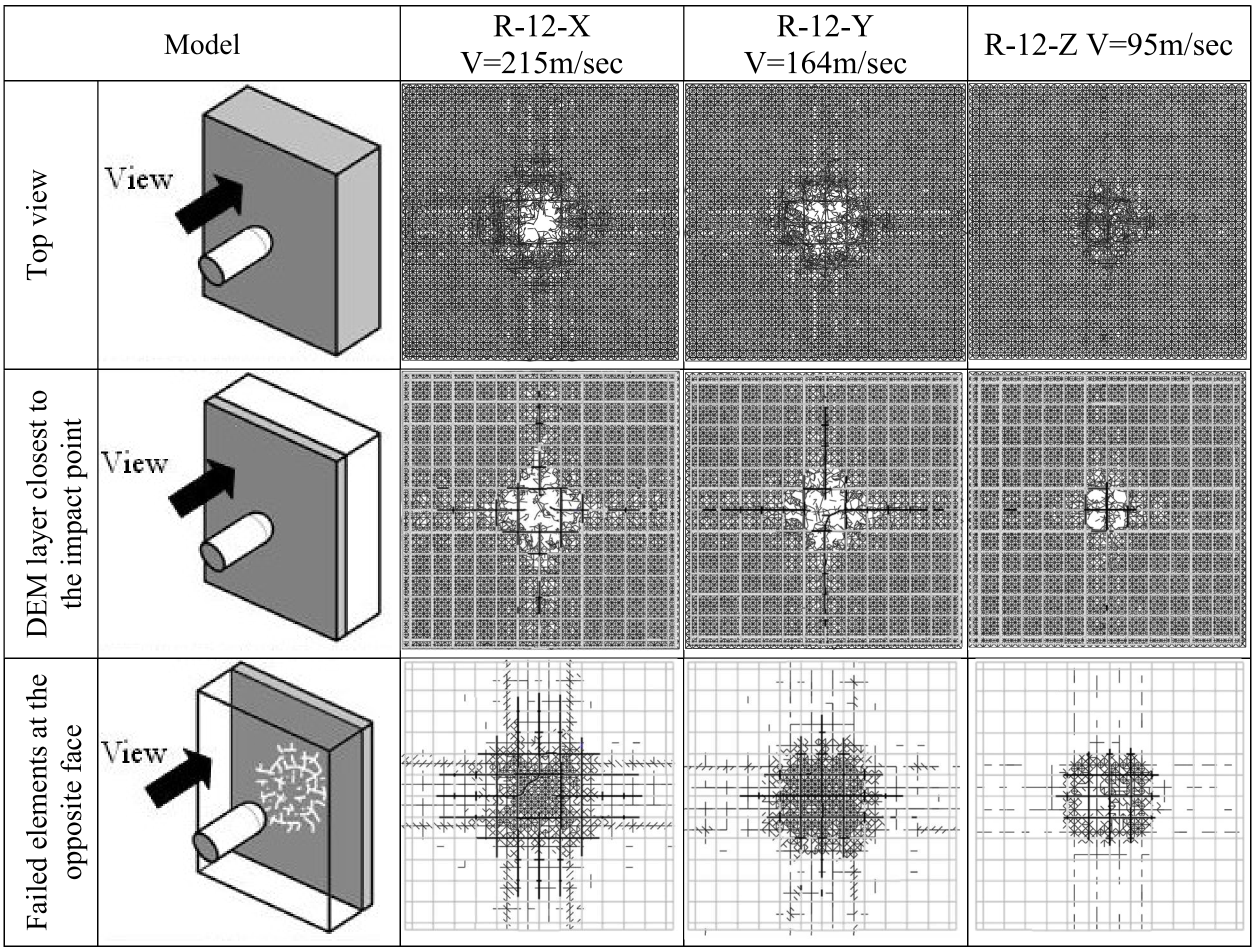

Figure 3. Numerical response prediction employing DEM model (Kosteski *et al.*, 2014)

PRESSURIZATION UP TO FAILURE OF REACTOR-CONTAINMENT MODELS

A series of tests of reactor containment models subjected to static internal pressure until failure have been conducted at Sandia National Laboratories over the past 20 years (Dameron *et al.*, 2002). The tests were designed to help understand the margin between design and failure pressures for different types of containment structures. The following large-scale models were tested to failure. The resulting information is considered valuable to furnish *lower bounds* on model error, because impact loading implies dynamic effects and certainly larger loading uncertainty than *static* internal pressure loading.

In this context, the following studies should be cited: (1) 1/8-scale model of a free-standing steel containment (typical of PWR ice-condenser steam-suppression systems) (test sponsored by the U.S. NRC);

(2) 1/6-scale model of a reinforced-concrete containment (typical of a large, dry PWR containment) (test sponsored by the U.S. NRC); (3) 1/10-scale model (with ¼-scaling of the shell thickness) of a steel BWR, Mark II containment (work sponsored by NUPEC and the U.S. NRC); and (4) 1/4-scale model of a pre-stressed concrete containment, modeled after Japanese PWR containments. (NUPEC and U.S. NRC joint project).

Knowledge of ultimate capacity is necessary to predict the response and potential consequences to a severe accident (Prinja *et al.*, 2005). From its inception, the containment programs have included pre- and post-test analyses, including so-called "round-robin programs" involving several international organizations. The results of these studies have typically shown considerable differences between numerical predictions and experimental results, mainly when the structural response becomes highly nonlinear and approaches failure, thus emphasizing the need for quantification of model uncertainty. In fact, very valuable data on model uncertainty of containment structures subjected to applied loadings was provided by the project described next. The containment system for the Pressurized Heavy Water Reactor (PHWR) of the 540 MWe Tarapur Nuclear Power Plant –CNT– (Units 3 and 4), India, consists of a primary containment building (internal) and a secondary vessel (external) that closes all systems and components, establishing an impermeable barrier against radioactive contamination of the environment (Singh, 2009). The physical model constructed at Bhabha Atomic Research Center (BARCOM) constitutes a detailed 1:4 scale representation of CNT's internal pre-stressed concrete containment building. The numerical model shown in Fig. 4a consists of Elastic Finite Elements, and a sector using nonlinear Discrete Elements to model concrete shown in Figure 4b, the steel reinforcement and the prestressing tendons. Details of the model were described by Kosteski *et al.* (2011). In Figure 4c the radial displacement in the position indicated by the red dot in the detail are depicted applied different pressure velocity in the simulations.

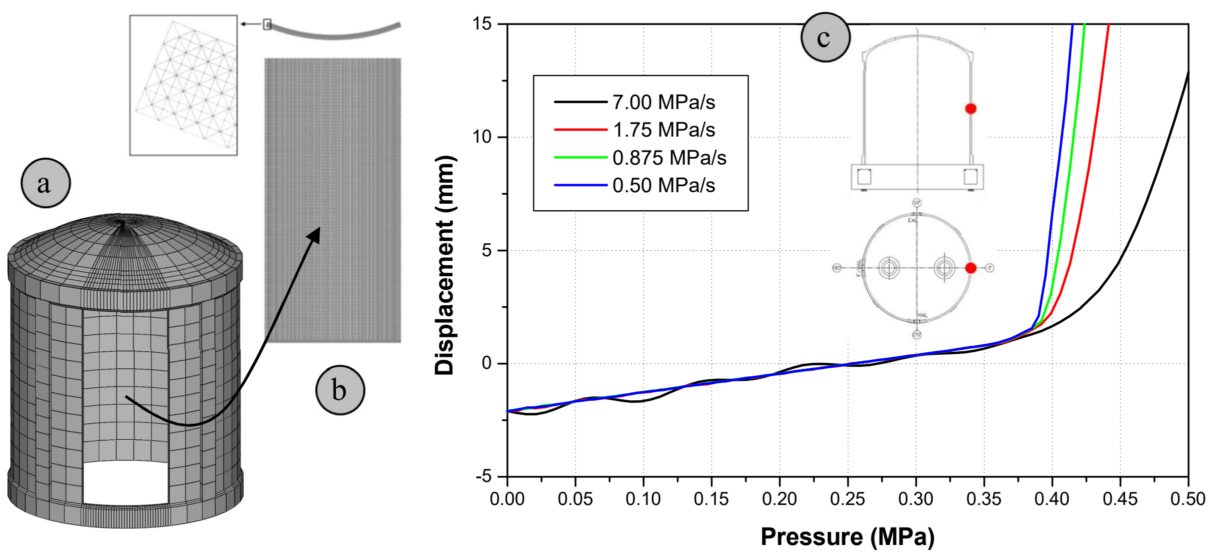

Figure 4: (a) Lateral and plan view of the model, (b) FEM+DEM numerical models, (c) response in terms of radial displacement vs. internal pressure for various loading rates Kosteski *et al.* (2011).

Figure 5 shows a comparison of the round robin predictions of the ten participants (Singh, 2009). All solutions employed nonlinear FEM models, except the thick gray line due to Kosteski *et al.* (2011), which employs the combination of DEM and FEM models described before. While for low pressures, say around 0.25 MPa, predictions differ by usual engineering tolerance, for pressure exceeding 0.25 MPa, when extensive fracture occurs, the dispersion of numerical predictions increases abruptly. The ultimate pressures estimated by the participants of the Round Robin project are presented in Table 3.

It may be seen that the mean value of the numerical predictions is 0.431 MPa, the standard deviation equals 0.077 MPa and the variation coefficient CV = 0.18. This value constitutes a preliminary estimate of model error in the use of large numerical models to determine structural loading capacity and is compatible with previous findings of round robin experiments for quasi-static loading, as exemplified by the CIGRE (1990) study on transmission towers subjected to wind loading, in which the CV of the wind ultimate load ranged between 0.17 and 0.35.

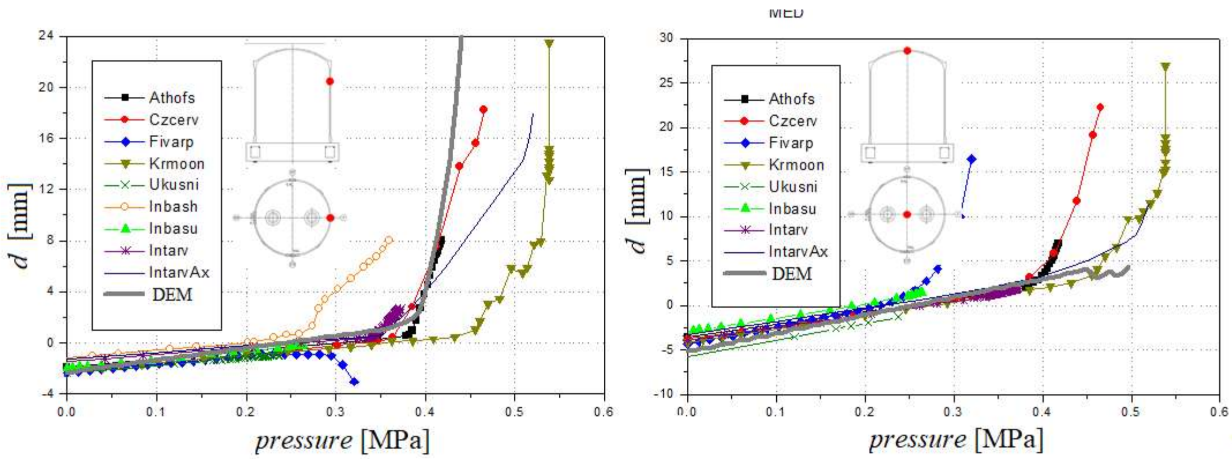

Figure 5. Comparison of numerical solutions (Normal displacement at locations identified by red dot vs applied pressure) FEM models except the LDEM+FEM model due to Kosteski *et al.* (2011), identified by the thick gray line (Singh, 2009) and as Participant #5 in Table3 below.

Table 3. Numerical predictions of ultimate internal pressure in BARCOM project

Participant	1	2	3	4	**5**	6	7	8
***P_u* (MPa)**	0.320	0.365	0.375	0.415	**0.448**	0.465	0.515	0.545

CONCLUSIONS

Since the birth of Structural Reliability, the uncertainty involved in the assessment of the applied excitation and of the structural system received almost all the attention of researchers and practicing engineers. In many cases, however, model uncertainty, that is, prediction errors resulting from the numerical model (FEM, DEM or others) and from the constitutive criteria adopted for the structural materials, size or rate effects, may largely exceed those due to the assumed excitation or structural properties.

The examination of model error in impact problems, presented in the paper, suggests that if the engineering prediction in any specific problem is considered a sample of a random variable and its mean value is estimated by the prediction, its coefficient of variation has a lower bound not inferior to CV= 0.10, determined for penetration equations. The corresponding CV for perforation equations, in view of the increased number of involved parameters, such as plate or shell thickness, reinforcement properties, etc. should logically be expected to be higher. Similarly, when large numerical models are employed to assess the loading capacity of structures subjected to impact, the coefficient of variation of the predicted capacity may be expected to exceed the value CV= 0.18, observed in several round robin experiments under *quasi-static* excitation.

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