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DISCUSSION OF NUMERICAL SIMULATION METHODS FOR IMPACT ANALYSES OF REINFORCED CONCRETE STRUCTURES

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ABSTRACT

One of the main objectives of the different phases of the benchmark project IRIS and of the research project IMPACT is to demonstrate and improve the capabilities of current finite element (FE) techniques to reproduce the detailed behaviour of a reinforced concrete (RC) structure under soft and hard impacts. The Nuclear Energy Agency (NEA) of the Organisation for Economic Co-operation and Development (OECD) organised the IRIS (Improving Robustness assessment methodologies for structures Impacted by miSsiles) project, whereas the IMPACT project was organised by the VTT Technical Research Centre in Espoo (Finland), and funded by several institutions, including the Swiss Federal Nuclear Safety Inspectorate ENSI. The IMPACT project so far has provided a comprehensive experimental database and information on physical phenomena of projectile impact and aircraft crash against RC structures. These findings contribute to calibration and improvement of modelling assumptions including adaptation of material models in the different FE programs (SOFiSTiK, Abaqus, and LS-DYNA) used for nonlinear dynamic simulation of the impact loading on RC structures. The objective of this paper is to assess the applied methods and the computer programs used as well as the experiences gained by numerical simulations of the tests in the above-mentioned projects.

INTRODUCTION

The mechanical behaviour of an RC structure subjected to an airplane impact is particularly affected by the stiffness of the impacting objects. The IMPACT test program, therefore, distinguishes between tests with predominantly flexural behaviour without pronounced local damage, tests with formation of a punching cone in combination with simultaneously developed bending cracks, and tests with punching behaviour leading to penetration and perforation of the RC structure.

In the tests on bending behaviour and on the investigation of combined bending and punching shear behaviour of slabs, deformable impacting bodies ("soft missile impact") replicate the impact of an aircraft fuselage or wing. The so-called “hard missile impact” tests on punching shear behaviour of slabs subjected to impact of non-deformable projectiles mainly represent the penetration and perforation of engines or other compact aircraft parts. A refined FE modelling of the target structure considering nonlinear material properties is required to obtain a detailed insight in the different characteristics of the structural response when reaching the ultimate load capacity.

Additional details regarding IRIS and IMPACT tests and their calculations mentioned in the present paper can be found in Rodríguez et al. (2013a, 2013b, 2014), Zinn et al. (2014), Borgerhoff et al. (2013, 2015, 2017), and Ghadimi Khasraghy et al. (2017, 2019a, 2019b).

NUMERICAL SIMULATION METHODS

Simulation of Impact Loaded RC Structures

The large number of numerical analyses carried out so far within the research project IMPACT have shown that modelling of RC slabs through nonlinear shell elements leads to a good simulation of the nonlinear stress-strain behaviour under the impact of deformable projectiles, see, for example, Borgerhoff et al. (2013). This statement applies to soft missile impacts on slabs showing exclusively flexural behaviour. In the combined bending and punching tests, nonlinear shell elements have demonstrated their suitability for a reliable numerical simulation of this problem, provided the slabs have a minimum shear reinforcement sufficient to ensure that the ultimate limit state related to punching shear is not exceeded. However, shell element models cannot provide any information about the strains of the shear reinforcement. Explicit modelling of the concrete with solid elements and the reinforcing bars with beam elements is required for this task. Three-dimensional modelling of the RC slab is essential if the projectile penetrates the compound structure of concrete and reinforcing bars, as is the case of hard missile impacts.

The numerical simulation of the impact of a deformable projectile may be carried out using a so-called “decoupled” analysis by applying a predefined impact load-time function on the target structure, and thus decoupling the interaction of the loading and the structural response. In this case, the load-time function is determined either using a simplified analytical approach such as Riera's (Riera, 1968), or using a separate nonlinear dynamic computation with an FE model of the projectile impacting a rigid target. In both cases, undeformable target structures are assumed for obtaining the load-time function. The decrease in loading due to interaction with the flexible structure is negligible for small deflections. The differences in the structural response resulting from a load-time function determined by means of FE analysis and the load according to the Riera’s approach with the same impulse are also small and usually negligible.

“Decoupled” analyses cannot be used in case of hard missile impacts. The penetration process of the RC slab can only be simulated with so-called “coupled” analyses of the impact loading, in which the interaction between the penetrating projectile and the RC components is taken into account.

Shell Element Modelling

The program used by the authors for the FE analyses with shell element models is SOFiSTiK; cf. SOFiSTiK (2022). The analysis of nonlinear effects in SOFiSTiK is done by iterations using a modified Newton method; i.e. an implicit integration scheme is used.

The RC target is modelled with shell elements in the SOFiSTiK simulations, which include nonlinear, layered concrete elements with approximate consideration of nonlinear shear deformations. The shell elements are subdivided into a predefined number of concrete layers, while a partition into 12 layers proved to be sufficiently accurate. The reinforcement layers on both sides take into account the reinforcing bars in both directions. The principal stresses are calculated at each concrete layer surface. Depending on the ratio of the two principal stresses in each principal stress direction, a fictitious uniaxial stress-strain law representing the biaxial behaviour is derived. The nonlinear stresses gained by that procedure are integrated across all layers to obtain the resulting forces and moments. Subsequently, the reinforcement forces, which include the tension stiffening effect, are added.

Moreover, the nonlinear behaviour of the RC shell elements in SOFiSTiK is defined by

- a nonlinear uniaxial stress-strain law of concrete considering compression-softening,
- consideration of tension softening of concrete after cracking as a function of the fracture energy,
- approximate consideration of transverse shear deformations by an elastic/perfectly plastic shear stress/shear strain law after exceeding the specified ultimate shear strength and
- a tri-linear stress-strain law of reinforcing steel with strain hardening.

Solid Element Modelling

The use of solid elements for the concrete, and beam elements for the embedded reinforcement provides detailed modelling of the RC target. In a “coupled” analysis, the finite element model of the projectile is also developed. Depending on the projectile design, shell elements (soft missiles) or solid elements (hard missiles) are used for modelling.

The authors have used the FE analysis programs LS-DYNA and Abaqus for the numerical simulation of impact scenarios with solid element models, cf. Livermore Software Technology Corporation (2020) and Simulia (2022). The problems were solved by explicit integration.

Different material models are chosen in LS-DYNA depending on the expected behaviour of the structure. The continuous surface cap model (material model 159) of LS-DYNA is used for the concrete when the peak response of the structure is of importance. This material model is mainly used for simulation of tests with punching behaviour, or those with a combined bending and punching response where the target perforation may be expected. This material model includes compression and tension softening of the concrete. Additionally, it allows for definition of an erosion criterion for the concrete. The Winfrith material model of LS-DYNA (material model 084/085) is used for the concrete when bending response of the target is expected. The concrete behaves elastic-perfectly plastic in compression not considering compression softening. Therefore, brittle compressive failure cannot be realistically simulated using this material law. The tension softening allows representing tensile failure with this material model.

The constitutive law used in LS-DYNA models for the longitudinal bars and stirrups is bilinear with strain hardening. The concrete and reinforcement elements are assumed to have a perfect bond, where the concrete solid elements are connected to the reinforcement beam elements at nodal points. The same nodes are defined for the reinforcement and the concrete where they are in contact with each other.

The constitutive law adopted for the concrete in Abaqus is the damaged plasticity model, which provides a general capability for the analysis of concrete under various types of loading. It includes a scalar damage model with tensile cracking and compressive crushing modes. The model accounts for the stiffness degradation associated with the irreversible damage that occurs in the fracturing process. When unloading from a post-peak situation, the load path does not necessarily return to the origin, giving rise to some permanent strains. For the reinforcement, an elastoplastic model is used in Abaqus. The impacting missiles are modelled using elastoplastic (Abaqus) or bilinear material models with strain hardening (LS-DYNA).

NUMERICAL SIMULATIONS OF IMPACT TESTS

Benchmark and Research Projects

The OECD/NEA benchmark project IRIS-2010 included two bending (soft impact) and three punching (hard impact) tests, which were carried out at the Technical Research Centre of Finland (VTT). The test specimens were square RC slabs with 2 m span supported on four sides. The slabs of the bending tests had thicknesses of 0.15 m, whereas the slabs of the punching tests were 0.25 m thick. The slabs were fixed to a steel frame, which was supported by four steel back-pipes, see Figure 1.

The bending test slabs B1 and B2 were designed to achieve a predominant flexural deformation behaviour and were reinforced with a large amount of shear reinforcement in order to prevent punching behaviour. On the other hand, the punching test slabs P1 to P3 were designed to fail in a punching mode and therefore did not contain any shear reinforcement.

Following IRIS-2010, the extensive experimental research program IMPACT was launched. As part of the research program IMPACT, which is organised by VTT and funded by several institutions, including the Swiss Federal Nuclear Safety Inspectorate ENSI, the execution of reduced-scale impact tests on reinforced concrete slabs started in Espoo (Finland). In the IMPACT phases I to III, in addition to the tests on bending and punching behaviour, a test series on combined bending and punching behaviour and another test series on vibration propagation and damping were carried out. The currently ongoing phase IMPACT IV is primarily dedicated to investigating whether the results of phases I to III can be transferred to target structures on a larger scale.

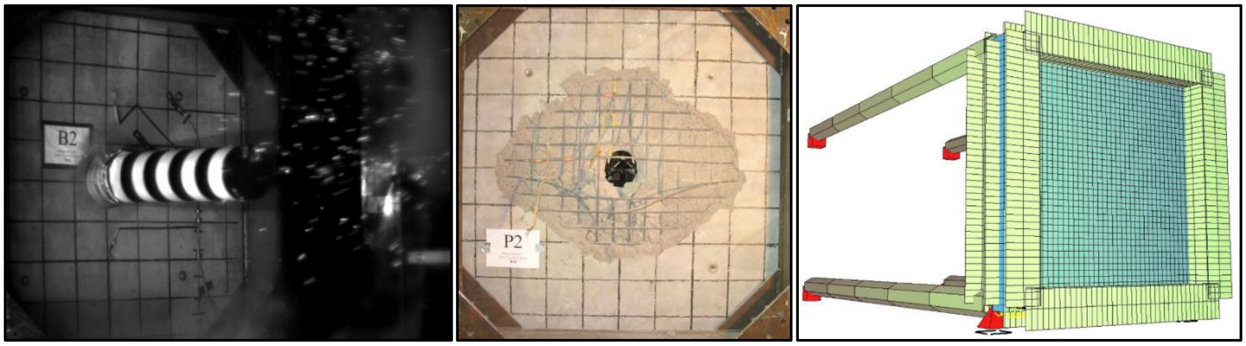

Figure 1. IRIS-2010 flexural test (left), rear side of punching test (centre), and a FE model (right)

Analyses of the Bending Tests

Analyses of tests with predominant flexural deformation behaviour have been carried out using the shell element model in SOFiSTiK program (Figure 1, right) within the IRIS-2010 benchmark project, cf. Borgerhoff et al. (2011). The load-time functions used for the computation were idealised functions derived from different Riera model solutions with varied assumptions; see Figure 2. The blind precomputation assumed the duration of the impact to be slightly less than 10 ms. Upon publication of the tested load duration of approx. 18 ms, the method of calculating the crushing resistance was improved by using the dynamic buckling resistance of a thin-walled circular tube acc. to Jones (2003). In addition, a load-time function defined as the contact forces between missile and target, which was derived by GRS within an explicit simulation using Ansys Autodyn software, was used for the post-computations. This improvement led to an excellent agreement of measured and calculated load duration and of measured and calculated displacement (and strain time) histories, see Figure 2. The influence of the oscillations of the GRS load function (probably correlated with a local buckling of the missile) on the calculated displacements, compared to the SPI load function on the computed results is obviously small.

Analyses of the Combined Bending and Punching Tests

As part of the IMPACT III project, the tests designed to promote the combined bending and punching response of the slabs (X test series) were carried out. Soft missiles with increased stiffness compared to the bending tests impacted reinforced concrete slabs at different velocities in the X test series, inducing both bending and punching shear behaviour of the slab.

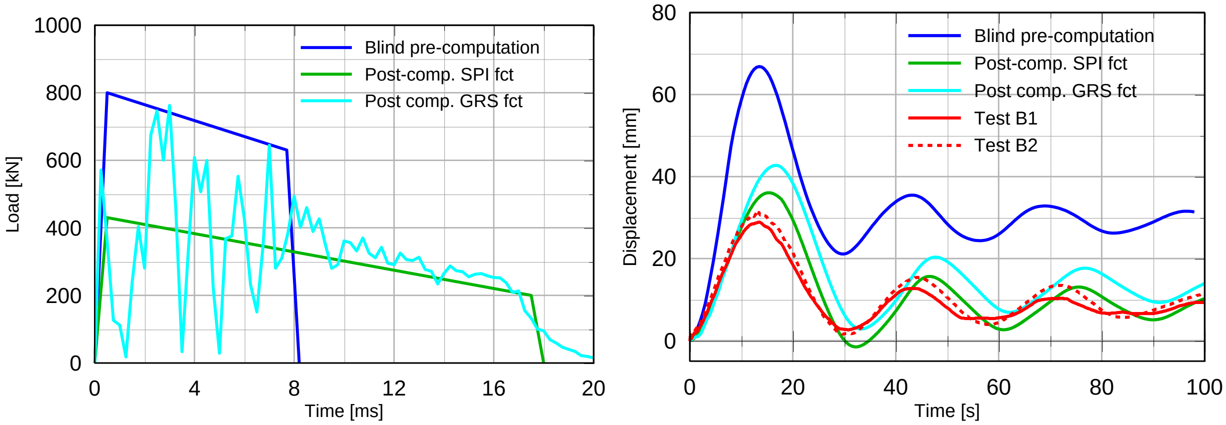

Figure 2. IRIS-2010 flexural test, load functions (left) and displacements of the slab centre (right)

The missiles were steel tubes with a total mass of usually 50 kg, subjected to predefined impact velocities ranging from 140 to 170 m/s. The targets were 250 mm thick reinforced concrete slabs with 2 m span supported on four sides. Except for one test, all RC target slabs contained shear stirrups in addition to the bending reinforcement. Calculations were conducted with Abaqus, SOFiSTiK, and LS-DYNA software.

The calculated impact load time history, defined as the contact forces between the missile of the test X3 (V=140 m/s) and a rigid wall obtained in Abaqus was later used in the SOFiSTiK calculations. The calculated load history compared to the curve obtained by Riera's equation in Figure 3 (left), shows a peak of about 3.7 MN and a total duration of 5.2 ms; the averaged values correlate reasonably well with the estimates using Riera's equation (Riera, 1968). The deformed missile after the impact is presented in Figure 3 (right). The calculated crushed length was about 430 mm, generating four folds in the shell; the test recorded a crushed length of 328 mm, with the same four concertina folds predicted by the calculations.

In the simplified shear stress/shear strain law used by the SOFiSTiK shell element, the punching shear resistance of the RC structure limits the magnitude of the shear forces. An important feature affecting this resistance is the angle of the punching cone, because it governs the activation of stirrups. The angle of a punching cone formed by an impact depends on many influencing parameters. In particular, the utilisation rate of the punching shear resistance, the ratio of concrete and stirrup contribution, the ratio of longitudinal and shear reinforcement, the deformability of the missile and the layout of transverse reinforcement should be mentioned. Based on the Abaqus solid element model computations and on the crack angles observed in test X3, which were induced by the positions of the stirrups, a mean punching cone angle of 45° was assumed (Borgerhoff et al., 2015).

The Abaqus calculation of the test X3 with the impact velocity of 140 m/s indicated that the missile does not perforate the slab. Having modelled successfully the impact at v = 140 m/s (test X3) in Abaqus, new calculations were conducted for increasing missile velocities, in increments of 10 m/s. The results for velocities up to v = 170 m/s are shown in Figure 4. The calculations showed that the perforation is still not expected at v = 160 m/s but will occur with a velocity of 170 m/s. In this situation, it was decided to conduct the X4 test at v = 165 m/s, a velocity for which an additional calculation was performed.

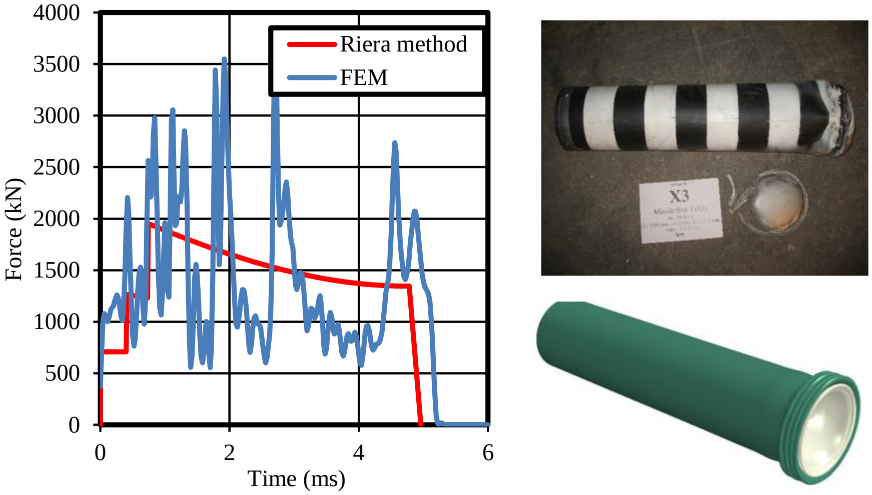

Figure 3. Load function for rigid wall (left), and deformed missile after the impact (right)

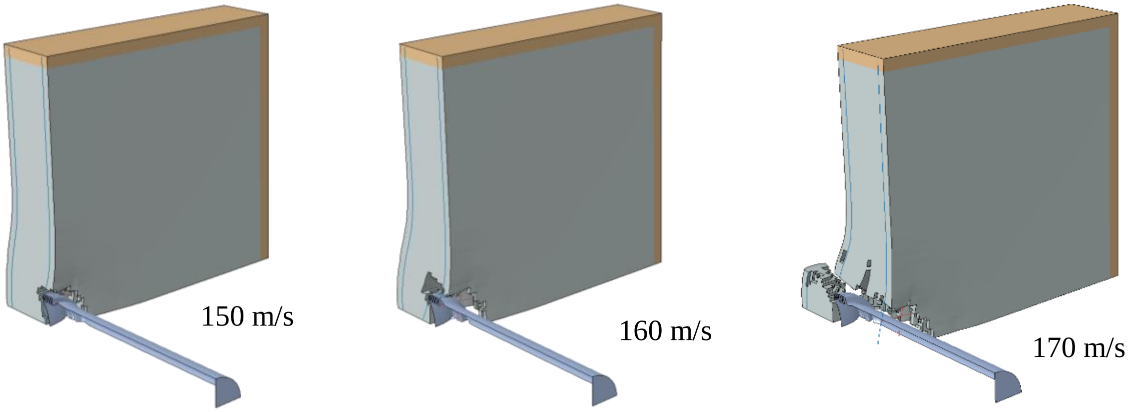

Figure 4. Deformation after 20 ms for different impact velocities

The simulations of test X4 were carried out with both Abaqus and LS-DYNA. The measured impact velocity of the test v = 168.6 m/s, is close to the ballistic limit of the slab. The Abaqus blind calculations predicted correctly that the missile would perforate the slab with a residual velocity of 22 m/s. The LS-DYNA post-calculation of the test also predicted a perforation with a residual velocity of 23 m/s, which is comparable to the measured residual velocity of 25 m/s during experiment. Figure 5 depicts the results obtained in the test and in the calculations.

The results of the further numerical analyses of the X test series are published in Borgerhoff et.al (2015, 2017) and Ghadimi Khasraghy et.al (2017, 2019a). Figure 6 shows, for example, the maximum slab deflections at the slab centre behind the loading area (sensor P1) for the tests X8, X9, and X10 calculated with SOFiSTiK and LS-DYNA. 250 mm thick slabs with different bending reinforcements ratios were subjected to impacts of 50 kg deformable projectiles with impact velocities of about 165 m/s. Deflections obtained from finite element models are in a good agreement with the measured data for tests X9 and X10. The measured values for Test X8 are likely to contain an error (they are much higher than another comparable test), perhaps due to loosening of the sensors.

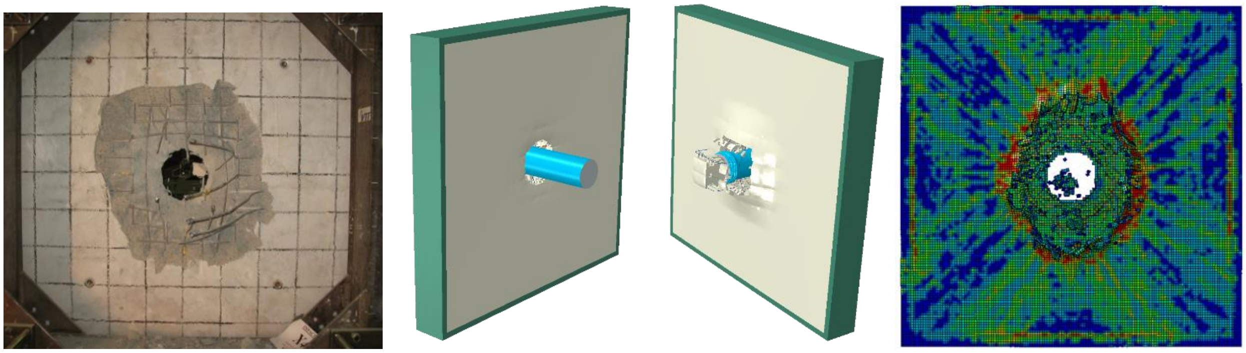

Figure 5. Rear view of X4 plate (left), Abaqus simulation (middle), and LS-DYNA simulation (right)

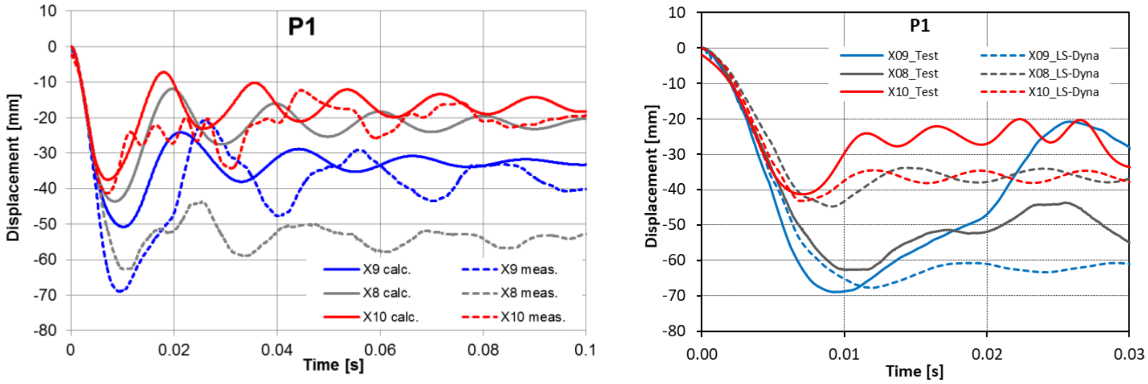

Figure 6. X8, X9, X10, slab centre displacements calculated with SOFiSTiK (left), LS-DYNA (right)

Analyses of the Punching Tests

As mentioned earlier, three identical punching tests were conducted as part of the Project IRIS-2010. Additional tests promoting punching behaviour of the RC slabs were performed as part of the IMPACT project. Numerical analyses of the IRIS-2010 punching tests were performed using explicit solver of Abaqus and LSDYNA software.

The three, almost identical, punching tests of IRIS-2010 perforated the slab with relatively high residual velocities of 34 – 45 m/s, which confirms the repeatability of these tests. Numerical simulations of the IRIS punching tests using LS-DYNA software lead to perforations with residual velocities of 44 – 63 m/s depending on the set of parameters (sensitivity analyses) used for defining erosion, as on the friction between the contacting surfaces of the missiles and the slabs. These velocities slightly overestimate the measured values.

The concrete damaged plasticity model was adopted for the Abaqus calculations, but it was slightly modified to achieve a better representation of the available triaxial tests. More specifically, the compressive cohesion stress was defined as a function, not only of the equivalent plastic strain, but also of the maximum principal stress. A finite element model was generated to represent the concrete, the reinforcing bars, and the missile. The rebars, with an elastoplastic constitutive model, were meshed with truss elements embedded in the concrete. In order to avoid excessive and unrealistic distortions, the elements were removed from the mesh when the equivalent plastic strain reached 0.3. This was accomplished with a user's subroutine. The stiffness contribution of the concrete becomes negligible when plastic strains exceed a value of 0.15, but a premature deletion of elements may produce inadequate results. Additionally, the Smoothed Particle Hydrodynamic (SPH) conversion technique was performed in some of the simulations with consistent results.

Figure 7 shows the damage in the rear side of the slabs after the impact (spalling and cracking) in comparison to the damage pattern after the IRIS P1 test. The damage patterns correspond to the experimental spalling and cracking. However, the extent of damage is slightly underestimated by the numerical analyses.

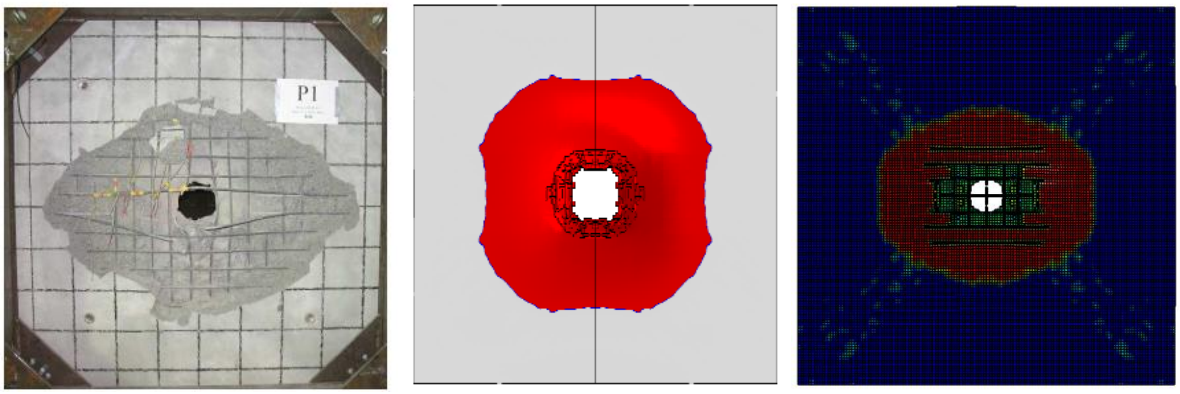

Figure 7. Slab damage, back view of the IRIS P1 (left), Abaqus model (middle), and LS-DYNA (right)

ASSESSMENT OF NUMERICAL METHODS AND DISCUSSION OF EXPERIENCES

Nonlinear dynamic finite element analyses were conducted for prediction of the behaviour of the reinforced concrete slabs subjected to impact loading as part of the IRIS and IMPACT projects. The authors used three different commercially available finite element software programs namely, SOFiSTiK, Abaqus, and LS-DYNA. Selected numerical results are compiled in the current paper.

The analyses using SOFiSTiK software included representation of the reinforced concrete using nonlinear layered shell elements. This numerical method, applying a predefined load-time history on the targets, provided good comparisons with the measurements for a wide range of tests when the bending behaviour of the slabs was dominant. The application was successfully extended to the tests promoting a combined bending and punching response of the slab, where the perforation of the slab was excluded.

The reinforced concrete was represented using solid elements for the concrete and beam elements for the reinforcements for the tests designed for a combined bending and punching response promoting slab perforations, as well as for the punching behaviour tests in which the slabs are subjected to hard impacts. The analyses using solid elements were conducted applying the explicit solvers of Abaqus and LS-DYNA programs. The finite element models of the missiles were also developed allowing “coupled” missile-target analyses, leading to a realistic representation of the missile penetration and tunnelling process. The analysis methods used by both software programs provided reliable numerical models simulating the punching response of the slabs subjected to deformable and non-deformable projectile impacts. SPH methods are additionally suited for modelling punching response of the RC structures.

Identifying the structural response phenomena (bending, punching, a combined response, or vibrations) to be simulated is important for choosing appropriate programs and element types, as well as for making the right modelling assumptions.

Irrespective of the software used, the material models representing the nonlinear behaviour of concrete should be chosen carefully. Numerical analyses are commonly conducted using available material models with "easily input" parameters in each software program. There are many of such integrated material laws available in different software. Therefore, careful assessment of the applied material model, its advantages and disadvantages, is vital for reasonable modelling of the RC structures. Different material models available in a commercial program may be suitable for different structural response phenomena. For instance, one material model may be better suited when the peak response of the slab is of interest (e.g. for punching tests), and another model may represent the cyclic and post peak response of the slab more reasonably (e.g. for bending tests). Therefore, a key to achieving reliable numerical models is selecting the most appropriate material model for the given problem depending on the expected slab response. Moreover, the material model should be calibrated with representative test results.

Sensitivity studies also contribute to the development of reliable numerical models. The selection of the main model parameters (e.g. element sizes, boundary conditions, failure criteria) should be validated using sensitivity studies.

For blind numerical predictions of the impact problems, it is recommended first to calibrate the main modelling parameters against other similar experimental data. The reliability of the numerical results, including dynamic nonlinear response of the RC structures subjected to the impact loading, is also analyst dependent. The experience of the analysts strongly contributes to making the right modelling assumptions considering the level of complexity of such nonlinear problems. Therefore, an independent review and validation of the analyses results is important.

CONCLUSIONS

Prediction of the nonlinear behaviour of the reinforced concrete slabs subjected to impact loading is a challenging task, which involves appropriate selection of the simulation programs and parameters. Three different commercially available finite element software programs namely, SOFiSTiK, Abaqus, and LS-DYNA, were used for simulation of the reinforced concrete slabs subjected to impact loading as part of the IRIS and IMPACT projects.

The current work explores the capability of numerical analysis to replicate the response of reinforced concrete slabs subjected to impact by deformable and non-deformable projectiles. The applicability of the analysis methods is discussed primarily considering the response of the impacted reinforced concrete structure, which depends both on its structural design and on the characteristics of the impacting projectile. In this context, the numerous non-linear dynamic FE analyses carried out as part of the IRIS and IMPACT projects are compared with the results of the experiments.

The impact of a deformable projectile can be modelled in a "decoupled" analysis using a predefined impact load-time function, obtained by assuming an impact on a rigid target. In case of hard missile impacts, the penetration process can only be simulated with "coupled" analyses that take into account the interaction between the penetrating projectile and the reinforced concrete structure.

Modelling of reinforced concrete slabs with layered shell elements leads to a good simulation of the nonlinear stress-strain behaviour under the impact of a deformable projectile when a punching failure is not expected. Crucial for the realistic modelling of the shear resistance in the case of combined bending and punching shear behaviour is a suitable assumption about the extent of the shear reinforcement activated during impact, which is largely determined by the punching shear angle. Explicit modelling of the concrete with solid elements and of the reinforcing bars with beam elements is essential if the projectile penetrates the RC structure, as occurs in hard missile impacts.

A large amount of basic data is available from the test results and calibration calculations carried out in the IRIS and IMPACT projects for the determination of the model assumptions to be made in design or verification calculations for impact-loaded reinforced concrete structures. The validation of the calculation models based on this experience leads to more realistic estimates of the failure limits and existing safety margins of nuclear power plants.

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