

NON-DIMENSIONAL APPROACH OF AIRCRAFT CRASH EQUATIONS AND APPLICATION TO PEAK LOADING FORCE

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INTRODUCTION

Engineers who are interested in evaluating the effects of aircraft crashes are very familiar with the Riera (1968) method, which is the reference method on the subject. The final output of the method is the impact loading force, $F(t)$, applied by the crashing aircraft on a rigid structure. In many situations, the interest of engineers is in evaluating the peak value of $F(t)$; however, the current engineering practice is to run the Riera method so as to calculate $F(t)$ and to retain the calculated peak value of it. The purpose of this paper is to develop a non-dimensional approach of Riera equations, which enables to derive an analytical evaluation of the peak loading force applied by a crashing aircraft on a rigid target, so that running transient analyses is not necessary anymore.

RECOLLECTION OF RIERA EQUATIONS

In Riera approach, an aircraft of mass M , length l_0 crashes on a rigid target with velocity v_0 . Considering that x is an abscissa measured from the tail, the aircraft mechanical description consists of

- aircraft linear mass density, dm/dx , exemplified in Figure 1a,
- integrated mass density $m(x)$, which is the mass of the part of aircraft comprised between the tail and the abscissa x . It is a monotonic increasing function such that $m(l_0)=M$ (Figure 1b).
- static buckling, or crushing, capacity of the aircraft fuselage at the abscissa x , $F_0(x)$, exemplified in Figure 1c. Its maximum value is denoted G_0 .

We introduce $c_0 = \sqrt{G_0 l_0 / M}$, which is a feature of the aircraft, homogeneous to a celerity.

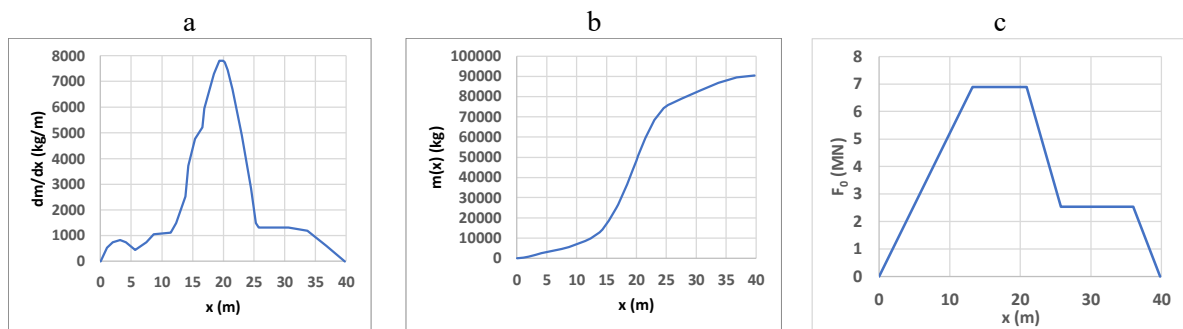


Figure 1. Examples of dm/dx , $m(x)$ and $F_0(x)$ function

Actual features of aircrafts are generally not available in literature. An exception is presented in the original Riera paper (1968) regarding the Boeing 720, reported in Figure 1. Although they might have been updated since 1968, we adopt them to support presentation of our rationale developments. Numerical values

are displayed in Appendix 1 (B720 is aircraft A1 in Table A1-1). Other aircraft features are presented in Appendix 1, and considered further in this paper. The range of impacting velocity, and particularly the maximum impacting velocity, depends on the type of aircraft. Realistic values of impacting velocities are also given in Appendix 1. In addition, it is worth noticing that, as well as above introduced features, the maximum value of dm/dx plays a role in the rationale.

The crash starts at time $t=0$. It is assumed that the aircraft structure destruction propagates backwards, from the nose to the tail of the fuselage. At a given moment, t , the front part of the aircraft is crashed and its velocity is zero, while the rear part is not yet crashed, not even deformed, and has a uniform velocity (see Figure 2). At this moment, the length of the rear part, measured from the tail, is designated by $x(t)$ and its velocity by $v(t)$. During the time lag between t and $t+dt$, an additional length dx of aircraft is crashed, such that $dx = -v dt$. Initial conditions are $x(0)=l_0$ and $v(0)=v_0$.

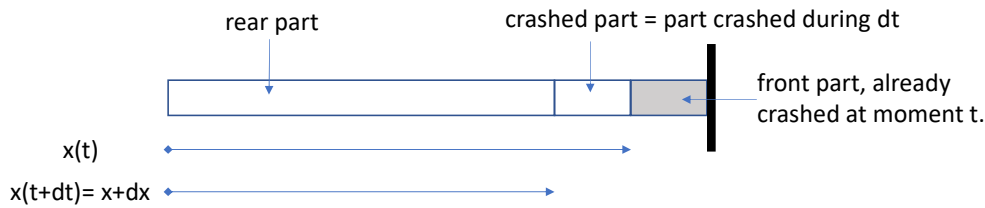


Figure 2. Notations

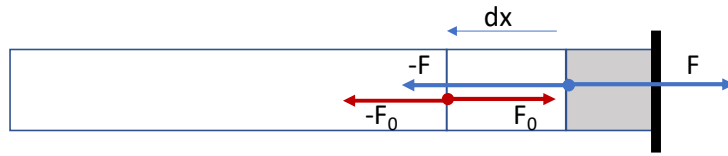


Figure 3. Forces applied on the crashed part

Riera assumption is that the force applied on the rear part, slowing it down, is the static fuselage capacity, $F_0(x)$. Concurrently, the loading force applied on the target is $F(t)$, as presented in Figure 3. Conservation of momentum for the rear part and the crashed part result in Riera equations, namely Equation 1 and Equation 2. (The dot on the top indicates derivation with respect to time.) In a first step, differential Equation 1 is resolved. In a second step, once $x(t)$ is identified, $F(t)$ is derived through Equation 2.

$$m(x) \ddot{x} = F_0(x), \text{ with } x(0)=l_0 \text{ and } \dot{x}(0) = -v_0. \quad (1)$$

$$F = F_0(x) + \frac{dm}{dx} (\dot{x})^2 \quad (2)$$

NON DIMENSIONAL EQUATIONS

We introduce the following non dimensional variables and functions:

- $y=x/l_0$;
- $h(x)=m(x)/M$;
- $f_0(x)=F_0(x)/G_0$, where G_0 is the maximum value of $F_0(x)$;
- $\mu(y)$ such that $\mu(y)=h(x)$, which implies: $m(x) = M \mu\left(\frac{x}{l_0}\right)$;
- $\varphi(y)$ such that $\varphi(y)=f_0(x)$.

For instance, the two non-dimensional functions $\mu(y)$ and $\varphi(y)$ of the above considered B720 are presented in Figure 4. As an effect of these new variables and functions, it results in particular the following:

- $\dot{y} = l_0 \dot{x}$ and $\ddot{y} = l_0 \ddot{x}$;
- $\frac{dm}{dx} = \frac{M}{l_0} \frac{d\mu}{dy}$.

Consequently, Riera equations turn into Equation 1' and Equation 2'.

$$\ddot{y} = \frac{1}{b_0} \frac{\varphi(y)}{\mu(y)}, \text{ with } b_0 = \frac{M l_0}{G_0}, y(0) = 1, \dot{y}(0) = -\frac{v_0}{l_0}. \quad (1')$$

$$\frac{F}{G_0} = \varphi(y) + b_0 \frac{d\mu}{dy} \dot{y}^2 \quad (2')$$

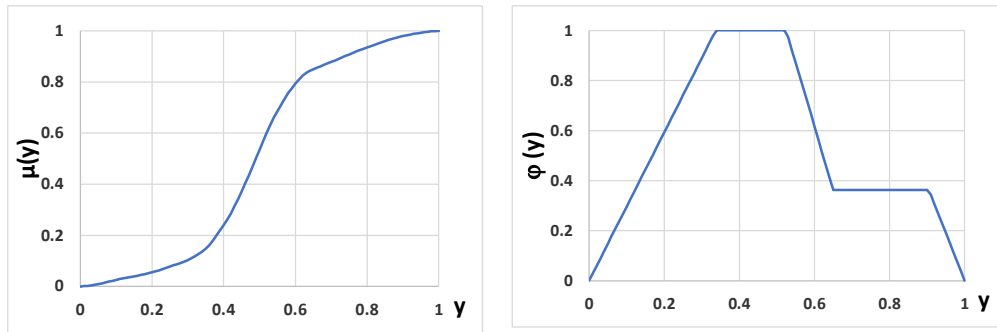


Figure 4. $\mu(y)$ and $\varphi(y)$ functions for the aircraft presented in Figure 1

It is possible to further simplify the case by introducing a non-dimensional time τ , and a new notation for derivation with respect to τ , as follows:

- $\tau=t/t_0$ with $t_0=l_0/v_0$,
- $y^\circ=dy/d\tau$, which results in $\dot{y} = \frac{1}{t_0} y^\circ$ and $\ddot{y} = \frac{1}{t_0^2} y^{\circ\circ}$.

With these new notations, Riera equations become Equation 1'' and Equation 2''.

$$y^{\circ\circ} = \frac{1}{a_0^2} \frac{\varphi(y)}{\mu(y)}, \text{ with } a_0 = \frac{v_0}{c_0}, y(0) = 1, y^\circ(0) = -1. \quad (1'')$$

$$\frac{F}{G_0} = \varphi(y) + a_0^2 \frac{d\mu}{dy} (y^\circ)^2 \quad (2'')$$

In a first step, differential Equation 1'' can be resolved, providing $y(\tau)$. Then $x(t)$ can be derived according to Equation 3. For given $\mu(y)$ and $\varphi(y)$ functions, the resolution is only dependent on a_0 , which appears as a non-dimensional velocity of the impacting aircraft. In practice, solutions of Equation 1'' presented in this paper were obtained through a central difference scheme with a non-dimensional time step $\Delta\tau=0.002$.

$$x(t) = l_0 y(t/t_0) \quad (3)$$

In a second step F/G_0 can be calculated as a function of τ through Equation 2'', and F derived as a function of t , as detailed in Equation 4. The non-dimensional loading force $\gamma(\tau)$ introduced in Equation 4 is discussed further.

$$F(t) = G_0 \gamma(t/t_0); \gamma(\tau) = \varphi(y(\tau)) + a_0^2 \frac{d\mu}{dy}(y(\tau)) (y'(\tau))^2 \quad (4)$$

EXAMPLES OF RESOLUTION

For the above introduced B720 $\mu(y)$ and $\varphi(y)$ functions, resolutions of Equation 1'' are presented in Figure 5 in the form of $y(\tau)$ and $y'(\tau)$, for $a_0=1.8$ and $a_0=3.2$. The corresponding non-dimensional loading forces $\gamma(\tau)$ are presented in Figure 6. The case $a_0=1.8$ ($v_0=100\text{m/s}$) is very close to the one calculated by Riera (1968) with $v_0=200$ knots (102 m/s). The $\gamma(\tau)$ peak value is equal to 10.27, resulting in a 70.9 MN peak loading force after it is multiplied by $G_0=6.9$ MN, while the peak value calculated by Riera is about 73 MN. The case $a_0=3.2$ gives a 34.52 $\gamma(\tau)$ peak value resulting in a 238 MN peak loading force.

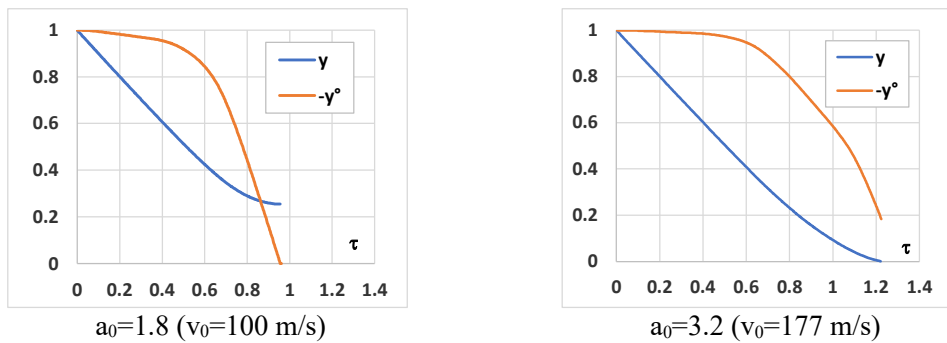


Figure 5: Resolution of Equation 1'' for aircraft A1 and two a_0 values

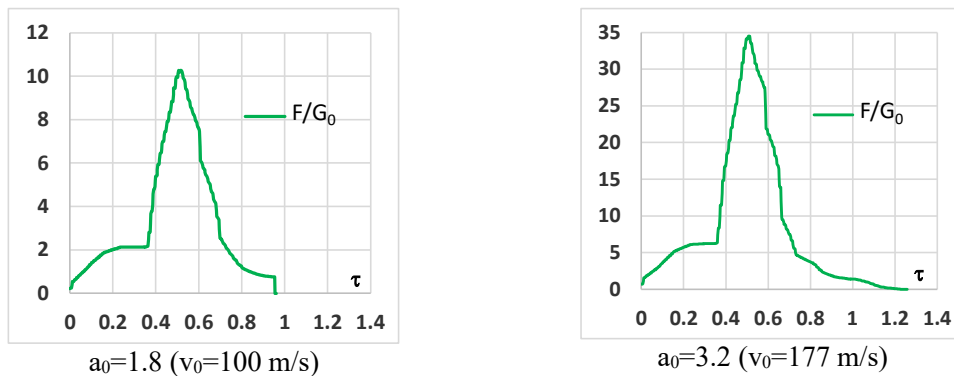


Figure 6: Non-dimensional loading force for aircraft A1 and two a_0 values

Another example is the case of a Phantom crash for $a_0=1.7$ ($v_0=225$ m/s), presented in Figure 7, calculated on the basis of features presented in Appendix 1. The $\gamma(\tau)$ peak value is equal to 7.28, resulting in a 131 MN peak loading force after it is multiplied by $G_0=18$ MN.

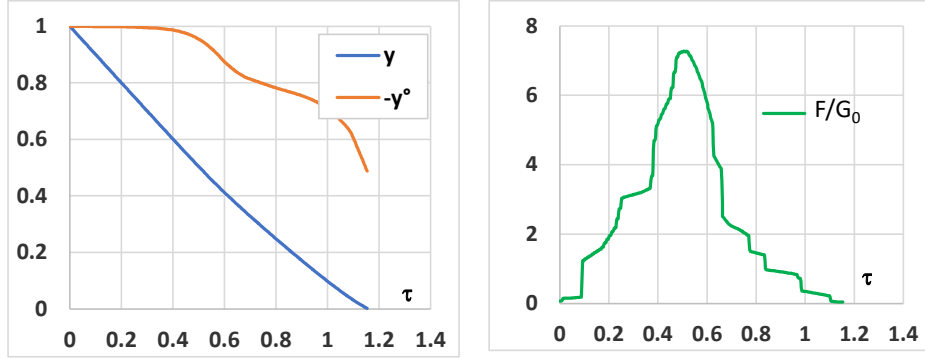


Figure 7: Resolution for the aircraft M1, with $a_0=1.7$ ($v_0=216$ m/s)

An interest of the non-dimensional approach is that $y(\tau)$ and $\gamma(\tau)$ responses of two different aircrafts with identical $\mu(y)$ and $\varphi(y)$ functions are identical. For instance, if we assume that B767-300 and B720 have identical $\mu(y)$ and $\varphi(y)$ functions, their non-dimensional responses are identical for a same a_0 value.

For the sake of brevity, we cannot present all outputs of analyses that were carried out for the 5 types of aircrafts presented in Appendix 1. Generally speaking, as exemplified by Figures 5 to 7, outputs of those analyses bring some comments:

- It can be observed in Figure 5 that for relatively small impact velocities, the calculation terminates by $y^\circ=0$ (the rear part velocity is decreased to zero), while for large impact velocities, it terminates by $y=0$, (the non-crashed rear part is decreased to zero). However, this later occurrence is very seldom when considering realistic impact velocities of commercial aircrafts given in Table A1-2.
- In all calculated cases, non-dimensional crash durations are very similar. For large impact velocities the non-dimensional crash duration is about 1.2, meaning that, as obviously expected, the crash duration is a little longer than the aircraft length divided by the initial velocity.
- In the realistic range of impacting velocities presented in Appendix 1, peak values of $\gamma(\tau)$ are significantly larger than 10 for aircrafts A1, A2, A3 and B1, but lower than 10 for M1.

PEAK LOADING FORCE

Our purpose is now to discuss the peak value of the loading force. It is controlled by the peak value, γ^{peak} , of the non-dimensional loading force, $\gamma(\tau)$, introduced in Equation 4. In a first step we present an overestimate, $\gamma_{\text{over}}^{\text{peak}}$ of γ^{peak} . In a second step, we introduce an approximate value of it, $\gamma_{\text{app}}^{\text{peak}}$, of which we establish the validity with reference to the calculated γ^{peak} values, for realistic situations as specified in Appendix 1.

Taking into account that both φ and y° cannot exceed 1, an overestimate of γ^{peak} reads obviously as given by Equation 5, and consequently an overestimate of the peak value of $F(t)$ is given by Equation 6.

$$\gamma_{\text{over}}^{\text{peak}} = 1 + a_0^2 \left(\frac{d\mu}{dy} \right)^{\text{peak}} \quad (5)$$

$$F_{\text{over}}^{\text{peak}} = G_0 + v_0^2 \left(\frac{dm}{dx} \right)^{\text{peak}} \quad (6)$$

However, it is possible to obtain a better approximation of the peak value. Of course, on the one hand, it should be expected that the peak or plateau values of φ and of $d\mu/dy$ are concurrent because they both correspond to the central part of the aircraft, where wings are attached to the fuselage. But on the other hand, it should be also considered that, although still close to 1, y° is not strictly equal to 1 at the moment when the central part of the aircraft crashes, because it is slightly slowed down by the front part crushing. Keeping also in mind that all calculated values of γ^{peak} are significantly larger than 1, we are led to propose an approximate of γ^{peak} as given by Equation 7.

$$\gamma_{\text{app}}^{\text{peak}} = a_0^2 \left(\frac{d\mu}{dy} \right)^{\text{peak}} \quad (7)$$

The validity of Equation 7 formula was examined for the 5 aircrafts and for the range of realistic a_0 values presented in Appendix 1. Outputs are presented in Figure 8, in the form of $\gamma_{\text{app}}^{\text{peak}}/\gamma^{\text{peak}}$ values versus a_0 . It is worth noticing that, at least for the considered cases, this approximate is still an overestimate for commercial aircrafts, the proposed approximate formula performing in particular very well for the B747 (Aircraft A2). The proposed approximate peak value slightly underestimates the peak force of the Phantom (Aircraft M1). This is relating to the above-mentioned fact that, compared to 1, the peak value of $\gamma(\tau)$ is not so large for the Phantom as it is for commercial aircrafts. Eventually, it should be mentioned that, the larger a_0 , the better the approximate value, in all circumstances.

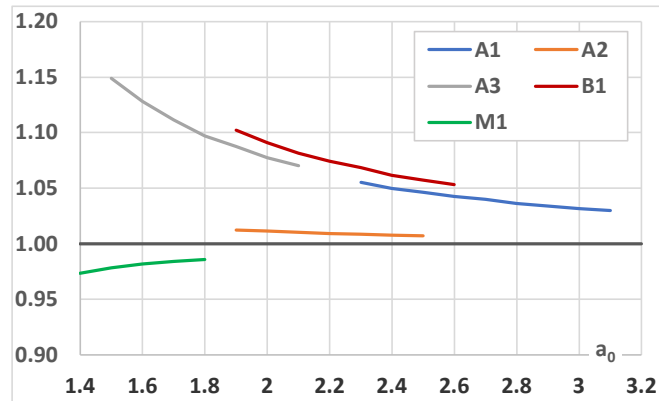


Figure 8: $\gamma_{\text{app}}^{\text{peak}}/\gamma^{\text{peak}}$ values versus a_0 for the 5 aircrafts presented in Appendix 1, in their respective ranges of realistic a_0 values

Consequently, with the same accuracy as reported in Figure 8, it is now possible to propose an approximate formula of the impact loading force peak value, presented in Equation 8. It is worth mentioning, as observed by engineers who are familiar with this type of analyses, that, according to this formula, the peak load is not dependent on the fuselage buckling or crushing capacity. Interestingly, it is also not dependent on the total mass of the crashing aircraft, while, of course, the integral of $F(t)$ is equal to the initial momentum of the aircraft, Mv_0 .

$$F_{\text{app}}^{\text{peak}} = v_0^2 \left(\frac{dm}{dx} \right)^{\text{peak}} \quad (8)$$

Generally speaking, in a similar manner to $\gamma(\tau)$, the larger the velocity, the better the approximate value. Taking the example of the B720 crashing at 100 m/s, the calculated peak force is 70.2 MN, while, based on $(dm/dx)^{peak}=7.800$ kg/m given in Appendix 1, the proposed Formula 8 results in 78 MN, a 11% exceedance. Yet, for the same aircraft crashing at 177 m/s, the calculated peak force is 238 MN, while the proposed Formula 8 results in 247 MN, a 4% exceedance.

As another illustration of Formula 8, we may consider the Phantom crashing at 225 m/s. The above calculated peak force is 131 MN, while, based on $(dm/dx)^{peak}=2550$ kg/m given in Appendix 1, Formula 8 output is 129 MN, an approximate value with a default smaller than 2%. As a matter of comparison, for a crash at 215 m/s, Formula 8 results in a 118 MN peak force, larger than the 110 MN given by the IAEA (2014). Having in mind that the IAEA retains a 2400 kg/m maximum value of dm/dx instead of 2550 kg/m, both outputs are perfectly consistent.

As last example, the Boeing 747, of which $(dm/dx)^{peak}=25000$ kg/m, generates a 250 MN peak force when crashing at 100 m/s, as predicted by Formula 8 as well as calculated by the IAEA (2014).

An interest of the non-dimensional approach is that, in the case of two different aircrafts with identical $\mu(y)$ and $\varphi(y)$ functions, it is possible to calculate the peak loading force generated by one of them from the peak loading force generated by the other one. For instance, assuming that a B767-300 ($M=180.000$ kg, $l_0=55$ m) has the same $\mu(y)$ and $\varphi(y)$ functions as a B720 ($M=90.000$ kg, $l_0=40$ m), and referring to the above calculated peak force of the B720 at 100 m/s (78 MN), we derive for the B767 at, say 160 m/s:

$$F_{app}^{peak,767,160} = \left(\frac{M}{l_0}\right)_{767} / \left(\frac{M}{l_0}\right)_{720} \left(\frac{160}{100}\right)^2 F_{app}^{peak,720,100} = 291 \text{ MN} \quad (9)$$

CONCLUSION

Non-dimensional analysis of aircraft crash equations, known in the engineering community as Riera equations, enables to substantiate that the crash phenomenon is controlled by two functions, $\mu(y)$ and $\varphi(y)$ that represent the mass distribution and the buckling (or crushing) capacity of the fuselage in non-dimensional terms, and one non-dimensional term, a_0 , which is a non-dimensional measure of the crashing aircraft velocity. It means that the impact loading forces of two different aircrafts with similar $\mu(y)$ and $\varphi(y)$ functions can be derived one from the other.

Regarding impact loading force peak values, the non-dimensional analysis provides evidence of the empirical rule that it is practically non-sensitive to the buckling capacity of the fuselage. It results that a reasonably accurate approximate formula is proposed (Equation 8) to evaluate the impact loading force peak value without necessity of running integration of aircraft crash equation at every encountered situation.

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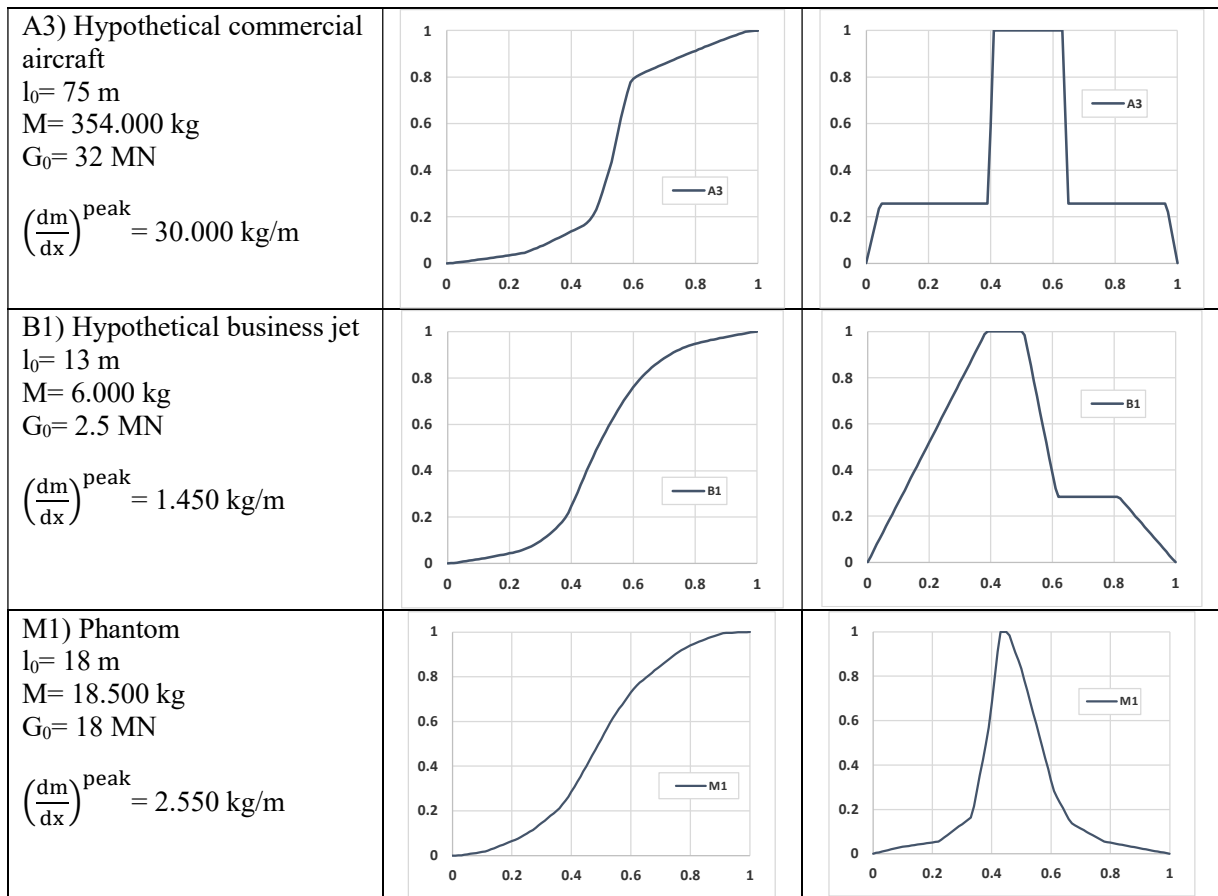
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APPENDIX 1
REPRESENTATIVE AIRCRAFT FEATURES

In addition to the description of Boeing 720 (aircraft A1 in Table A1-1) given by Riera (1968), a description of the Boeing 707, of which the Boeing 720 is representative, is given by Wolf et al. (1978). The IAEA Safety Report 87 (2014), provides a description of $m(x)$ and $F_0(x)$ functions of the Boeing 747 (aircraft A2 in Table A1-1). In the same table, A3 is a third realistic hypothetical commercial aircraft, while B1 is a realistic hypothetical business jet. Regarding an example of military aircraft (aircraft M1 in Table A1-1), the Phantom mass distribution is presented by von Rieseman et al. (1989), while a simplified version of it is given by the IAEA (2014); our input data are based on von Rieseman et al. (1989), retaining an original Phantom aircraft. Considering the buckling or crushing force, Muto et al. (1989) presents two possible representations of $F_0(x)$; we selected a compromise between them, with an 18 MN estimated G_0 value.

Table A1-1: Features of considered aircrafts

Aircraft Identifier	$\mu(y)$	$\varphi(y)$
A1) Boeing 720 $l_0 = 40$ m $M = 90.000$ kg $G_0 = 6.9$ MN $\left(\frac{dm}{dx}\right)^{peak} = 7.800$ kg/m		
A2) Boeing 747 $l_0 = 68,6$ m $M = 378.000$ kg $G_0 = 25$ MN $\left(\frac{dm}{dx}\right)^{peak} = 25.000$ kg/m		



Regarding crashing velocity of commercial aircrafts, its maximum value is often assumed to be around 150 m/s; we consider here the 130-170 m/s range. For military aircraft, the IAEA (2014) indicates 215 m/s as an appropriate value; we consider here the range 180-240 m/s. Without specific information on it, we assume 150-200 m/s for the business aircraft. These realistic values, as well as c_0 and the corresponding retained range of a_0 values are summarized in Table A1-2. For numerical applications, the a_0 path is 0.1.

Table A1-2: Range of realistic impacting velocities and corresponding a_0 values

Aircraft	c_0 (m/s)	Impacting velocities (m/s)	a_0
A1	55.4	130-170	2.3 - 3.1
A2	67.4	130-170	1.9 - 2.5
A3	82.5	130-170	1.5 - 2.1
B1	76.2	150-200	1.9 - 2.6
M1	132.3	180-240	1.4 - 1.8