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A METHODOLOGICAL APPROACH TO UPDATE GROUND MOTION MODELS USING BAYESIAN INFERENCE

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ABSTRACT

In recent decades, prediction of ground motion at a specific site or a region is of primary interest in probabilistic seismic hazard assessment (PSHA). Historically, several ground motion prediction equation (GMPE) models with different functional forms have been published using strong ground motion records available from NGA-West and European databases. However, low-to-moderate seismicity regions, such as Central & Eastern United States and western Europe, is characterized by limited strong-motion records in the magnitude-distance range of interest for PSHA. In these regions, the available data for the development of empirical GMPEs is very scarce and limited to small magnitude events. For these regions, the general practice in PSHA is to consider a set of GMPEs developed from data sets collected in other regions with high seismicity. This practice generates an overestimation of the seismic hazard for the low seismicity regions. There are two potential solutions to overcome this problem: (i) a new GMPE model can be developed; however, development of such a model can require significant amount of data which is not usually available, and (ii) the existing GMPE models can be recalibrated based on the data sets collected in the new region rather than developing a new GMPE model. In this paper, we propose a methodological approach to recalibrate the coefficients in a GMPE model using different algorithms to perform Bayesian inference. The coefficients are recalibrated for a subset of European Strong-Motion (ESM) database that corresponds to low-to-moderate seismicity records. In this study, different statistical models are compared based on the functional form given by the chosen GMPE, and the best model and algorithm are recommended using the concept of information criteria.

INTRODUCTION

The expected ground motion at a site is represented by an attenuation relationship or a ground motion prediction equation (GMPE). GMPEs estimate the intensity of ground shaking and their underlying uncertainty based on ground motion characteristics such as earthquake magnitude, propagation path and soil conditions, style-of-faulting, and other seismological parameters (Chandrakanth et al., 2019). In recent years, many methodologies (Bertin et al., 2020; Bodda et al., 2022; Kowsari et al., 2020; Wang and Takada, 2009) based on Bayesian inference have been used to update the model parameters by accounting for various sources of uncertainty. Wang and Takada (2009) updated an existing GMPE model for site-specific ground motion records based on K-NET and KiK-net databases. The functional form of the original GMPE is considered as fixed and a linear correction term function of magnitude and distance is added to it. The unknown parameters are estimated using analytical formulations and the predictions obtained from the calibrated model are found out to be unbiased. In a recent study, Bertin et al. (2020) calibrated nine existing GMPEs against RESORCE-2013 database. The GMPE models are considered as fixed and a bias term is added to the GMPE functional form. The bias term and the variance of GMPE model are estimated using Metropolis Hastings algorithm. Kowsari et al. (2019) updated seven GMPEs of different functional forms based on strong-motion PGA data from the South Iceland Seismic Zone. The posterior probability density function of all regression parameters are estimated using Metropolis within Gibbs Markov Chain Monte Carlo (MCMC) sampler and the priors are chosen as non-informative. More recently, Kowsari et al. (2020) further improved upon Kowsari et al. (2019) and updated four GMPEs at various periods ranging from $0.05s$ to $5s$. In Kowsari et al. (2020), informative priors are used to update higher order regression parameters of GMPE, and Delayed Rejection and Adaptive Metropolis (DRAM) algorithm is implemented for efficient sampling of highly correlated regression parameters.

In the aforementioned studies (Bertin et al., 2020; Kowsari et al., 2019, 2020; Wang and Takada, 2009), the original GMPEs are calibrated for new data set either by (i) fixing the GMPE functional form and estimating additional terms to account for bias correction, or (ii) estimating all the regression parameters in the GMPE model. The former approach is easy to implement and the bias correction terms can be estimated using analytical formulations. The second approach to estimate all the regression parameters in a non-linear GMPE functional form can be challenging, computationally demanding, and it requires careful selection of MCMC algorithms. Moreover, the non-linearity in a generic GMPE functional form which is a function of moment magnitude (M_W) and distance (R) terms usually comes from a regression coefficient (h) that represents the pseudo-depth parameter (see Equation 1). Therefore, the performance of the recalibrated GMPE model with h being a fixed (linear functional form) parameter versus h not being a fixed (non-linear functional form) parameter must be studied.

$$\mathbb{E}\left[\log y\right] = b_0 + \left(b_1 + b_2 M_W\right)\log\left(\sqrt{R^2+h^2}\right) + b_3 M_W + b_4 M_W^2 \tag{1}$$

where, $\log y$ is the logarithm of the ground motion intensity measure of interest, M_W is the moment magnitude, R is the distance. $b_0, b_1, b_2, b_3, b_4, h$ are the coefficients of the GMPE model.

In this research, we propose a Bayesian methodology to update GMPE internal parameters or coefficients for ground motion records corresponding to a new region. The parameters are estimated using different algorithms and the effect of prior information (weakly informative vs informative priors) on the GMPE predictions is evaluated. In this study, different statistical models are compared based on the functional form given by the chosen GMPE and the best model is selected using several information criteria for European Strong-Motion (ESM) database.

PROPOSED METHODOLOGY

The proposed methodology employs four key stages that are described below:

Stage 1: Assessment Base

- Develop an assessment base of ground motion records for a region or site-specific location of interest. Some of the widely used databases around the world are RESORCE (Akkar et al., 2014), ESM (Lanzano et al., 2019), NGA-West2 (Ancheta et al., 2014), NGA-East (Goulet et al., 2014), K-NET and KiK-net (NIED, 2019) databases.

- For a given application domain of interest, select a subset of records from the complete database based on earthquake characteristics.

- Divide the selected subset records randomly into two data sets: (i) training data set – for updating the coefficients of GMPE, and (ii) testing data set – for testing the performance of the recalibrated GMPE.

Stage 2: Evaluation Models

- Select GMPE models from the existing literature that are applicable for the target seismotectonic context.

Stage 3: Model Updating

When a GMPE model calibrated on a particular data set predicts response on a new unseen data set, some kind of discrepancy always exists between the observed response and the predicted response. The process of estimating this discrepancy (or bias correction) and any unknown parameters in the model is known as inverse uncertainty quantification (Nagel, 2019; Wu et al., 2018). In the proposed methodology, the following types of statistical models are updated and subsequently compared in terms of information criteria:

- *Fixed parameter setting with added bias*: In this setting (Equation 2), the functional form of original GMPE model is fixed and a bias correction term μ_k is added. Since $f_{BI14}(x_i)$ is fixed, the unknown parameters that need to be estimated are $\theta = \left(\mu, \sigma^2\right)$.

$$\log y_i \quad = \quad \log\left(10^{f_{BI14}(x_i)+\mu-2}/g\right) + \varepsilon_i, \qquad \varepsilon_i \sim N(\mu, \sigma^2) \tag{2}$$

- *Uncertain parameter setting:* In this setting (Equation 3), the functional form of the GMPE model is not fixed and one or more parameters in the model are updated based on the new observations. The unknown parameters that needs to be estimated are $\theta = \left(\eta, \sigma^2\right)$.

$$\log y_i \quad = \quad \log\left(10^{f_{BI14}(x_i,\eta)-2}/g\right) + \varepsilon_i, \qquad \varepsilon_i \sim N(0, \sigma^2) \tag{3}$$

  where, η is a vector of regression coefficients (aka parameters) in BI14.

Stage 4: Assess Evaluation Model Adequacy

- Compare/rank the updated statistical models using information criteria (described in section **??**).
- Test the performance of the recalibrated GMPE models using the testing data set.

GMPE FUNCTIONAL FORM AND DATABASE

In this study, we have selected the GMPE by Bindi et al. (2014) (referred to as BI14 in the rest of the manuscript) and the functional form of the GMPE model is shown below:

$$\begin{aligned} \log y_i &= \log\left(10^{f_{BI14}(x_i)-2}/g\right) + \varepsilon_i \\ &= f_{BI14}(x_i)\log 10 - 2\log 10 - \log g + \varepsilon_i \end{aligned} \tag{4}$$

where, y_i is the i-th ground motion intensity measure of interest (PGA or pseudo spectral acceleration (PSA) at a given period in terms of g) of the considered database, $i = 1, \ldots, n$, and $f_{BI14}(x_i)$ is the intensity measure predicted by BI14, given a vector x_i of characteristics of the earthquake; $\varepsilon_i \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$ is a stochastic error term, representing the variability of BI14 prediction with respect to observed records. The deterministic part of the GMPE model f_{BI14} is given in Bindi et al. (2014).

In this study, we use a subset of ESM database to update the BI14 coefficients. The selection criteria for moment magnitude [3; 5.2], distance [1; 350 km], $V_{s,30}$ velocity [800; 1200 m/s], and fault mechanism [normal, strike-slip and reverse] is chosen such that the selected records correspond to low-to-moderate seismicity records such as French territory (Traversa et al., 2020). This resulted in a subset of 3154 observations, out of which 2200 records are used for updating the BI14 coefficients and 954 records are used for testing the performance of the recalibrated BI14 model.

ESTIMATION METHODS AND INFORMATION CRITERIA

The concept of Bayesian inference has been widely used in many applications to estimate unknown parameters with uncertainty (Bodda et al., 2020a,b, 2021; Keller et al., 2014; Vaishanav et al., 2020; Viallet et al., 2017). Bayesian inference is a type of statistical inference method in which the probabilities or distribution of uncertain parameters are updated as new evidence or information becomes available. The posterior density is computed using Bayes theorem (Reich and Ghosh, 2019):

$$p\left(\theta|D\right) = \frac{f(D|\theta)\pi(\theta)}{m(D)} \propto f(D|\theta)\pi(\theta) \tag{5}$$

where,

$$\begin{aligned}
D &= \left(x_1, x_2, \ldots, x_n\right), \text{ vector of observed data} \\
\pi(\theta) &= \text{prior density of } \theta \\
f(D|\theta) &= \text{Likelihood or sampling density of } D \text{ given } \theta \\
m(D) &= \int f(D|\theta)\pi(\theta)d\theta, \text{ marginal density of } D \\
p\left(\theta|D\right) &= \text{posterior density of } \theta \text{ given } D
\end{aligned}$$

Except in very simple cases, $m(D)$ is not available in closed-form, making the posterior density intractable. That is why, in many cases widely used numerical methods such as Markov Chain Monte Carlo (MCMC) methods or Importance sampling (IS) methods are employed to generate many posterior samples which approximates the true posterior distribution. MCMC and IS are a family of methods, rather than one particular algorithm. Some of the well-known MCMC algorithms are Metropolis-Hastings algorithm, Gibbs sampling, Slice sampling etc. (Reich and Ghosh, 2019). The sampling importance resampling (SIR) algorithm is one of the most commonly applied IS methods to nonlinear Bayesian problems (Heine, 2005; Keller et al., 2014). The various estimation methods used for Bayesian update of GMPE parameters in this research are: (i) Bayesian Linear Models (BLM) using Conjugate Priors, (ii) Sampling Importance Resampling (SIR) with Laplace Approximation, (iii) Adaptive Metropolis (AM), and (iv) Automated factor slice sampling (AFSS). In this study, deviance information criteria (DIC) and Watanabe–Akaike (or widely applicable) information criteria ($WAIC$) is used for model comparison.

APPLICATION

In this study, three statistical models (M1, M2, M3) based on the functional form of the BI14 GMPE are considered (see Equation 6).

$$\begin{aligned}
&\text{M1: } \log y_i = \log\left(10^{f_{BI14}(x_i)+\mu_{BI14}-2}/g\right) + \varepsilon_{BI14,i}, \quad \theta_{M1} = (\mu_{BI14}, \sigma_{BI14}) \\
&\text{M2: } \log y_i = \log\left(10^{f_{BI14}(x_i,\eta)-2}/g\right) + \varepsilon_{BI14,i}, \quad \theta_{M2} = (\eta, \sigma_{BI14}) \\
&\text{M3: } \log y_i = \log\left(10^{f_{BI14}(x_i,\eta,h)-2}/g\right) + \varepsilon_{BI14,i}, \quad \theta_{M3} = (\eta, h, \sigma_{BI14})
\end{aligned} \tag{6}$$

M1 refers to the fixed parameter setting with added bias term. M2 refers to the uncertain parameter setting with coefficient h being fixed. Therefore, M2 becomes a linear model with a linear magnitude-dependent distance scaling. The term "linear" corresponds to the linearity in the GMPE coefficients $\eta = (e_1, c_1, c_2, c_3, b_1, b_2, \gamma, f_1, f_2)$ but not in terms of the GMPE explanatory variables such as magnitude, distance, $V_{s,30}$, and style-of-faulting. M3 is same as M2 with h not being fixed, making it as a nonlinear model. In both M2 and M3, coefficient b_3 is not updated because the training data set does not contain data that has magnitude M_W more than hinge magnitude $M_h = 6.75$. Also, coefficient f_3 is not updated as the third fault class (F_S strike-slip) or categorical variable is redundant. Coefficients c_3 and h are constrained to be non-negative.

Parameter Updating

The peak ground acceleration (PGA) and 5% damped pseudospectral acceleration (PSA) for periods $T = 0.2s, 1s$ are computed using OpenQuake Engine (Pagani et al., 2014) and GMPE-SMTK toolkit (Weatherill et al., 2014). The PGA and PSA observations are obtained by calculating the geometric mean of the two horizontal components. The parameters are estimated/recalibrated using four algorithms: BLM, SIR, AM, and AFSS. BLM algorithm is implemented only for models M1 and M2. For models M1, M2, and M3 the parameters are first estimated using weakly informative priors: $\beta_{BI14} \sim \mathcal{N}(0, 10^2)$, $\mu_{BI14} \sim \mathcal{N}(0, 10^2)$, $\sigma^2_{BI14} \sim \mathcal{IV}(a = 0.001, b = 0.001)$.

For M2 and M3, the parameters are also recalibrated based on informative normal-inverse gamma priors. The prior for regression coefficients is a normal distribution with mean equal to their original value and variance equal to the total variance of BI14 (Table 1). The samples from MCMC (AM; AFSS) algorithms are obtained by considering two parallel chains with (100,000; 5000) iterations for each chain and a thinning interval of (50; 2). A thinning interval of t means selective saving of every t^{th} sample. Thinning is applied to reduce the autocorrelation between MCMC samples. In MCMC, the samples from the initial iterations do not represent the target posterior density, referred to as the burn-in period. Therefore, these samples are discarded for the subsequent analysis. In this study, a burn-in of 500 iterations is applied for the output MCMC samples after thinning. The first chain is initiated using the original values of regression coefficients and the second chain is initiated by specifying all the parameter values equal to 0.1 (random initial values can also be chosen). In Laplace approximation, the mode $\tilde{\theta}$ of posterior distribution $p(\theta|D)$ is estimated using Trust Region optimization method (Statisticat and LLC., 2020). Next, 10,000 random samples are generated using the Laplace approximated posterior distribution. In SIR, the posterior density of parameters is estimated from the above samples by resampling the samples with a size equal to the effective sample size (ESS) and probabilities proportional to the sample weights.

The results of the recalibrated BI14 regression coefficients estimated using various algorithms for models M2 and M3 are tabulated Table 1, respectively. For comparison, the original coefficients of the BI14 are also provided in this table along with the mean and standard deviation (given in brackets) obtained from the posterior distribution of regression coefficients. The BLM approach estimates the regression coefficients without any approximations where as the results obtained from the MCMC and SIR algorithms are based on approximations. The accuracy of the algorithms are compared in the following Section in a formal way using the concept of information criteria. The coefficient h that is causing non-linearity in M3 is examined as shown in Table 1. The recalibrated value of h is very close its original value indicating that the new data set does not have much effect on the estimation of coefficient h. Therefore, the predictions estimated using non-linear model M3 can be similar to the predictions estimated using linear model M2. As observed in Table 1, the standard deviation of coefficients is greater for uninformative priors, compared to the informative priors. Therefore, it is better to use informative priors for GMPE parameter recalibration. We also see an increase in total sigma σ_T of the recalibrated GMPE compared to the original BI14 GMPE. This is because the BI14 GMPE is originally derived from the RESORCE database whereas the ESM database comprises of records from RESORCE database, and many other additional records from the pan-European region. Therefore, increasing amount of data implies increasing the spatio-temporal diversity of ground motion records, and thus an increase in total sigma σ_T for the recalibrated BI14 GMPE on the ESM training database (Kotha et al., 2020).

Table 1: Comparison of original BI14 coefficients with M2 and M3 recalibrated coefficients for PSA $(T = 1s)$ – *the standard deviation of coefficients are given in brackets*

Coefficients	Org.	*M2: Linear*	*M3: Non-Linear*		
		BLM	**SIR**	**AM**	**AFSS**
e_1	3.1247				
Uninformative		2.63 (0.224)	3.7075 (0.469)	3.19 (0.277)	3.2189 (0.243)
Informative		3.5769 (0.148)	3.2299 (0.253)	3.1023 (0.14)	3.2383 (0.248)
c_1	-1.0527				
Uninformative		-1.0379 (0.095)	-1.039 (0.111)	-0.9946 (0.071)	-0.9877 (0.076)
Informative		-0.7448 (0.082)	-0.9902 (0.079)	-1.0104 (0.052)	-0.9904 (0.078)
c_2	0.1035				
Uninformative		0.2357 (0.061)	0.2377 (0.063)	0.2589 (0.083)	0.2764 (0.045)
Informative		0.4395 (0.051)	0.2751 (0.047)	0.2775 (0.044)	0.274 (0.048)
c_3	0				
Uninformative		7e-4 (2.8e-4)	6.5e-4 (3.3e-4)	7.5e-4 (4.2e-4)	7e-4 (2.7e-4)
Informative		0.001 (2.8e-4)	6.8e-4 (2.8e-4)	5.5e-4 (2e-4)	6.9e-4 (2.8e-4)
b_1	0.3066				
Uninformative		0.9674 (0.295)	0.9628 (0.302)	0.5825 (0.193)	0.6252 (0.174)
Informative		-0.1853 (0.194)	0.6328 (0.176)	0.5032 (0.063)	0.6385 (0.172)
b_2	-0.1476				
Uninformative		0.1012 (0.054)	0.1011 (0.055)	0.0271 (0.047)	0.0437 (0.039)
Informative		-0.0643 (0.039)	0.0449 (0.039)	0.0185 (0.019)	0.0455 (0.038)
γ	-0.8266				
Uninformative		-1.2179 (0.191)	-1.2146 (0.191)	-1.1872 (0.192)	-1.1275 (0.168)
Informative		-1.0783 (0.166)	-1.1209 (0.166)	-1.1216 (0.189)	-1.1236 (0.168)
f_1	0.0263				
Uninformative		-0.0034 (0.021)	-0.003 (0.021)	0.0067 (0.05)	-0.0037 (0.022)
Informative		0.0004 (0.022)	-0.0026 (0.021)	-0.0018 (0.024)	-0.0025 (0.022)
f_2	0.0186				
Uninformative		-0.081 (0.027)	-0.0808 (0.027)	-0.0622 (0.08)	-0.081 (0.027)
Informative		-0.0794 (0.027)	-0.0811 (0.027)	-0.0811 (0.03)	-0.0801 (0.027)
h	4.4161				
Uninformative		–	4.5108 (1.205)	4.5272 (0.999)	4.4313 (0.339)
Informative		–	4.4215 (0.337)	4.4644 (0.316)	4.4102 (0.332)
σ_T	0.3561				
Uninformative		0.8672 (0.151)	0.87 (0.013)	0.8644 (0.054)	0.8697 (0.013)
Informative		0.8842 (0.154)	0.8698 (0.013)	0.8699 (0.013)	0.87 (0.013)

Model Selection

In this section, the statistical models M1, M2, and M3 are compared based on their goodness of fit and model complexity using information criteria. Table 2 shows the DIC and $WAIC$ scores for all the different types of models and estimation methods considered in this study. The $WAIC$ scores for M2 and M3 are greater than their DIC scores when the parameters are estimated using AM algorithm. $WAIC$ is evaluated by averaging over the posterior distribution where as DIC is estimated by conditioning on a point estimate (Gelman et al., 2014). As there is a lot of uncertainty in the predictions estimated using AM algorithm due to bad convergence, $WAIC$ captures the overall uncertainty and gives a high score. The $WAIC$ and DIC scores are close to each other for M2 and M3 when the parameters are estimated using BLM, SIR, and

AFSS algorithms. Therefore, it is better to select the linear model (M2) rather than non-linear model (M3) for Bayesian recalibration of parameters. Also, BLM algorithm is preferred over AFSS or SIR algorithms because BLM provides results without approximations and is computationally efficient for large data sets. If the non-linear functional form of the GMPE is considered for Bayesian recalibration of coefficients, then SIR algorithm can be employed for better computational efficiency.

Table 2: Comparison of DIC and $WAIC$ scores

Model	Estimation Method	PGA		PSA (0.2s)		PSA (1s)	
		DIC	$WAIC$	DIC	$WAIC$	DIC	$WAIC$
M1: Added-Bias	BLM	6712.30	6713.18	6742.08	6743.01	5987.16	5988.64
	SIR	6716.35	6716.25	6746.15	6746.08	5991.22	5991.71
	AM	6722.23	6721.19	6749.61	6750.59	5992.52	5993.08
	AFSS	6716.59	6716.43	6746.07	6746.00	5991.18	5991.69
M2: Linear	BLM	6139.22	6140.53	6322.09	6323.29	5633.46	5635.42
	SIR	6140.92	6141.92	6324.39	6325.11	5633.72	5635.73
	AM	6168.59	6171.24	6669.41	9611.83	5793.69	6608.39
	AFSS	6150.25	6144.02	6326.97	6325.25	5636.17	5630.25
M3: Non-Linear	SIR	6135.53	6136.56	6321.81	6322.55	5633.96	5636.01
	AM	6403.54	8584.04	6704.76	9917.72	5768.62	6378.53
	AFSS	6142.98	6136.03	6323.77	6323.94	5638.72	5608.87

Testing

In the final step of the proposed methodology, the performance of the recalibrated GMPE is tested against the testing data set $(\tilde{x}, \tilde{y})$. The testing data set is not used in the recalibration of GMPE parameters. Figure 1 shows the root mean squared error (RMSE) estimates obtained from different functional forms of BI14: M1, M2, and M3 considered in this study along with the original functional form of BI14 (M0) for comparison.

As seen in Figure 1, the model predictions has improved compared to the original BI14 predictions when the added bias term is added. It has improved further when the coefficients of BI14 are recalibrated using the training data set.

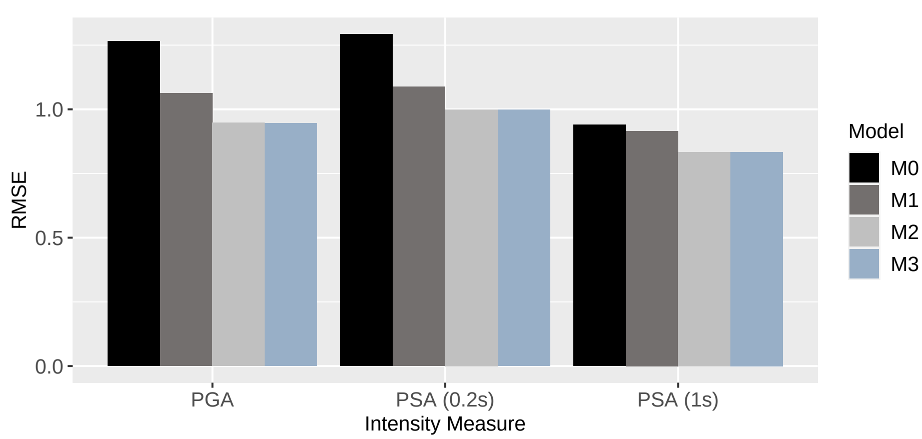

Figure 1. Comparison of RMSE statistic for testing data set

SUMMARY AND CONCLUSIONS

In this manuscript, we have introduced a Bayesian methodology to recalibrate the coefficients/parameters of a GMPE model for ground motion records corresponding to a new region. Three statistical models (added bias, linear, and non-linear) based on different function forms of BI14 GMPE are considered for Bayesian update of parameters. In this research, a subset of ESM database that corresponds to low-to-moderate seismicity region is considered. The selection criteria is chosen as [3; 5.2], [1; 600 km], [800; 1200 m/s], and [normal, strike-slip and reverse] for moment magnitude, distance, $V_{s,30}$ velocity, and fault mechanism, respectively. The subset records are divided into training and testing data sets, where the training data set is used for recalibration of parameters and the testing data is used for testing the performance of different GMPE functional forms. The parameters are updated using Bayesian linear models (BLM), Sampling importance resampling (SIR) with Laplace approximation as a proposal distribution, and in addition to SIR, two MCMC algorithms: Adaptive Metropolis (AM) and Automated factor slice sampling (AFSS) are considered. BLM algorithm is only applicable for models M1 and M2, and the parameters are estimated without approximations.

The coefficient h that is causing non-linearity in M3 is also examined. The recalibrated value of h is found out to be almost same as the original value. Therefore, the predictions from models M2 and M3 must be in a similar range. To confirm this, we compared the DIC and $WAIC$ information criteria scores between models. The scores are similar to each other for M2 and M3 when the parameters are estimated using BLM, SIR, and AFSS algorithms. Therefore, the linear functional form (M2) of BI14 can be used for parameter recalibration with BLM algorithm for computational efficiency. If one uses the non-linear functional form of the GMPE, then SIR algorithm can be utilized for recalibration of regression coefficients.

Finally, the performance of the models is tested against a testing data set and root mean squared error (RMSE) is used as a measure for comparing the performance. The Bayesian recalibration of parameters (M2 and M3) have improved the RMSE compared to the original BI14 model and added bias model (M1). The reduction in RMSE is significant for PGA and lower time period ($T = 0.2s$) giving confidence in the proposed methodology of Bayesian update of GMPE parameters.

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