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# VIBRATION TEST AND FATIGUE TEST FOR FAILURE PROBABILITY EVALUATION METHOD WITH INTEGRATED ENERGY

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## INTRODUCTION

The seismic evaluation of key components such as reactor vessel is important for the Seismic Probabilistic Risk Assessment (S-PRA) in a Sodium-Cooled Fast Reactor (SFR). Many components were damaged by cumulative damage like fatigue damage during seismic ground motion. However, general evaluation method for key components under seismic ground motion has been based on static loads and elastic region of materials. More accurate evaluation method for S-PRA, which can evaluate the failure of key components such as reactor vessels, has been actually required. The correlation between fatigue failure and the integrated vibration energy to components was already confirmed in the previous study by Minagawa et al. (2009). Furthermore, the fatigue failure evaluation and fragility curve with the vibration energy were investigated by Minagawa et al. (2015). In this study, failure probability evaluation method with integrated energy was developed by comparing the energy with vibration tests and fatigue tests. Vibration tests were performed to evaluate integrated vibration energy at failure by energy balance equation and fatigue tests were performed to evaluate integrated vibration energy at failure based on experimental results of fatigue tests.

## ENERGY BALANCE EQUATION

The energy balance equation of a single degree of freedom model is as follows,

$$m\int_0^t \ddot{x}\dot{x}dt + c\int_0^t \dot{x}^2 dt + \int_0^t F(x)\dot{x}dt = m\int_0^t \ddot{z}_h \dot{x}dt \tag{1}$$

where, $x$ is a displacement, $m$ is a mass, $c$ is a damping coefficient, $F(x)$ is a restoring force, $\ddot{z}_h$ is an acceleration of ground motion. The first term, second term and third term of left hand side of the equation showed kinetic energy, dumping energy and elastic strain energy and cumulative plastic strain energy, respectively. Right hand side of the equation showed total integrated energy or input integrated energy. The total integrated energy of left hand side of the equation is equal to the input integrated energy of right side of the equation after the vibration. Accordingly, the input integrated energy is evaluated as the integrated energy of the model.

## VIBRATION TEST

The vibration tests with the simple test specimens, whose dimension is shown in Figure 1, were carried out by vibration table. Picture of test specimen is shown in Figure 2. The specimen has two arms (right side and left side) and mass of 130 g is set at end of the arms. Thickness of the arm is 1 mm, width is 10 mm and length is 150 mm. Frequency of the specimen is 5.3 Hz. Wave form of the tests is controlled as sine wave and the maximum input accelerations are 15m/s$^2$, 22m/s$^2$ and 30 m/s$^2$. Picture of fractured test specimen is shown in Figure 3.

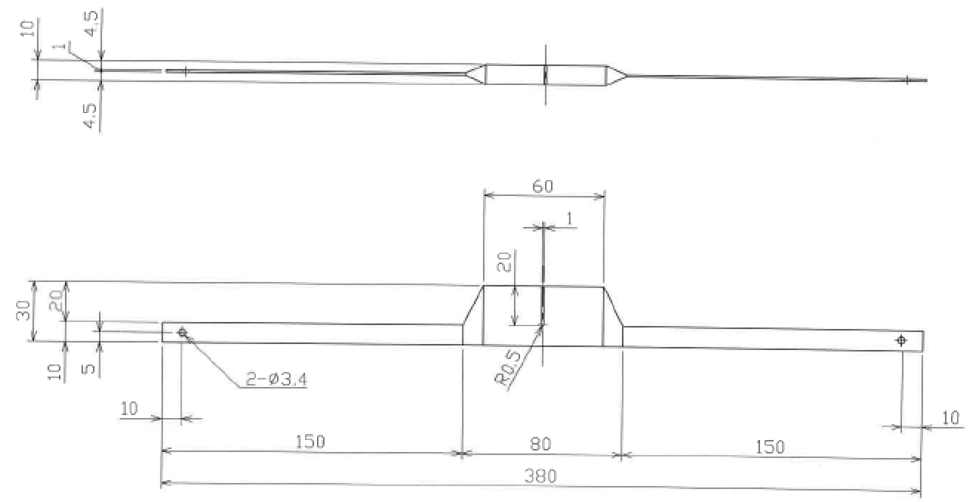


Figure 1. Dimension of specimen for vibration test.

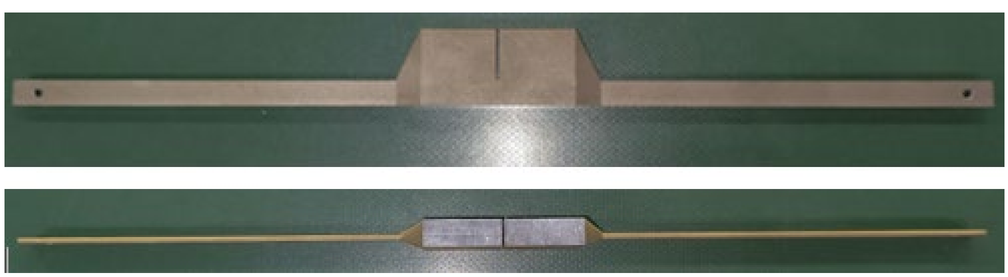

Figure 2. Picture of test specimen for vibration test.

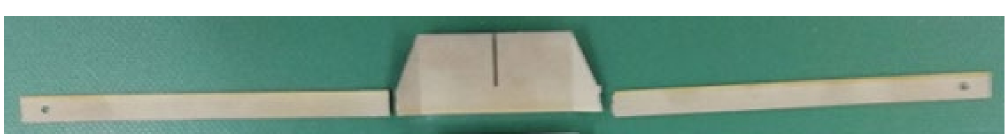

Figure 3. Picture of fractured test specimen for vibration test.

As results of vibration test, the integrated energy at failure is estimated by right hand side of equation (1), and estimated values are listed in Table 1. The integrated energy at failure were among 3,000 J and 6,000 J. Mean of integrated energy at failure was about 4,700 J and standard deviation was about 1,000 J. Relationship between integrated energy at failure and input acceleration was shown in Figure 4.

Table 1: Input acceleration, failure time and integrated energy of test specimen

| Input acceletion | Left arm | | Right arm | |
|---|---|---|---|---|
| | Time | Integrated energy | Time | Integrated energy |
| [m/s$^2$] | [sec.] | [J] | [sec.] | [J] |
| 15 | 3019 | 2990 | 4020 | 4287 |
| | 3337 | 3209 | 3815 | 3948 |
| 16 | 4442 | 4509 | 4613 | 4699 |
| 22 | 2635 | 5086 | 2505 | 4825 |
| 23 | 3220 | 5942 | 3020 | 5555 |
| 30 | 1781 | 4815 | 1367 | 3733 |
| | 2128 | 6123 | 2083 | 6068 |

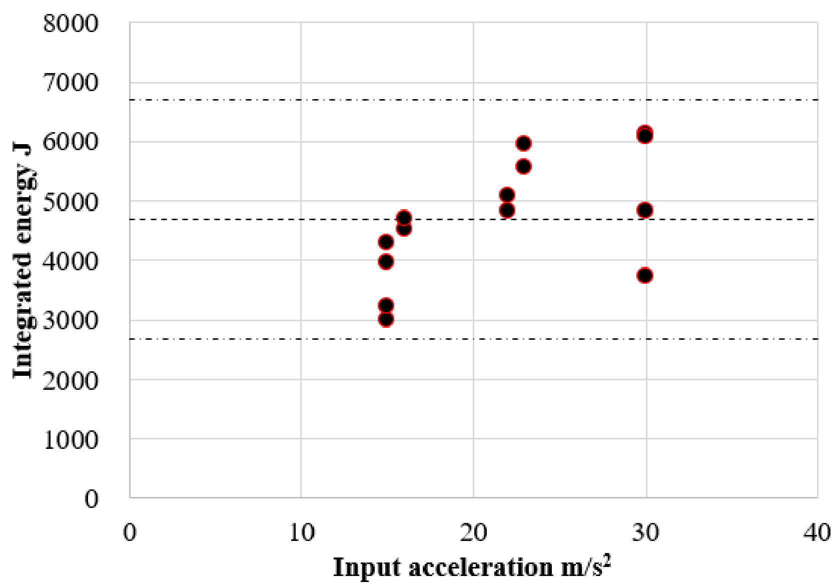


Figure 4. Relationship between integrated energy at failure and input acceleration.

## FATIGUE TEST OF MATERIAL

The aim of fatigue test is to confirm relation of stress amplitude and number of cycles to failure to estimate integrated energy at failure without vibration tests. The fatigue tests were carried out with round-bar test specimens under a condition of stress control. Dimensions of a test specimen and appearance are shown in Figure 5 and 6, respectively. Parallel part of specimen is hand polished along axial direction to remove scratch by lathe machine. Wave form and its frequency are controlled to be sine wave and 0.1 Hz, respectively. Fatigue test machine (Shimadzu Corporation; EHF-UG100kN-20L) is shown in Figure 7.

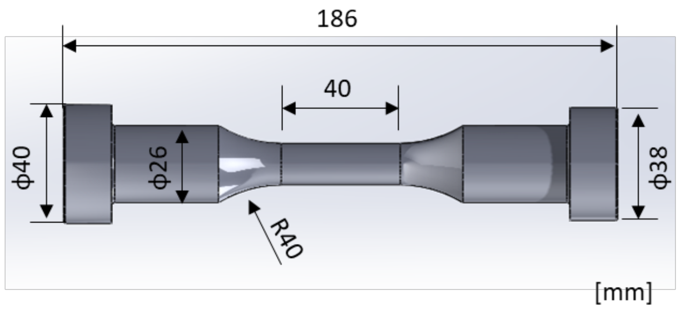


Figure 5. Dimensions of a test specimen.

Figure 6. Appearance of a test specimen.

Figure 7. Appearance of a fatigue test machine.

As the results, stress amplitude and number of cycles to failure are listed in Table 2. The relationship between stress amplitude and number of cycles to failure is shown as S-N curve in Figure 8 and the equation for stress amplitude vs number of cycles to failure was approximated up to the stress amplitude of 450 MPa. The equation is as follows,

$$S_a = -43.06ln(N_f) + 724.49 \quad (2)$$

where, $S_a$ is stress amplitude and $N_f$ is number of cycles to failure.

Table 2: Stress amplitude and number of cycles to failure

| Specimen No. | Stress amplitude [MPa] | Number of cycles to failure [ - ] |
|---|---|---|
| 1 | 455.4 | 287 |
| 2 | 456.0 | 706 |
| 3 | 423.1 | 1,799 |
| 4 | 390.9 | 2,594 |
| 5 | 325.4 | 11,192 |
| 6 | 293.6 | 19,105 |
| 7 | 293.2 | 17,414 |

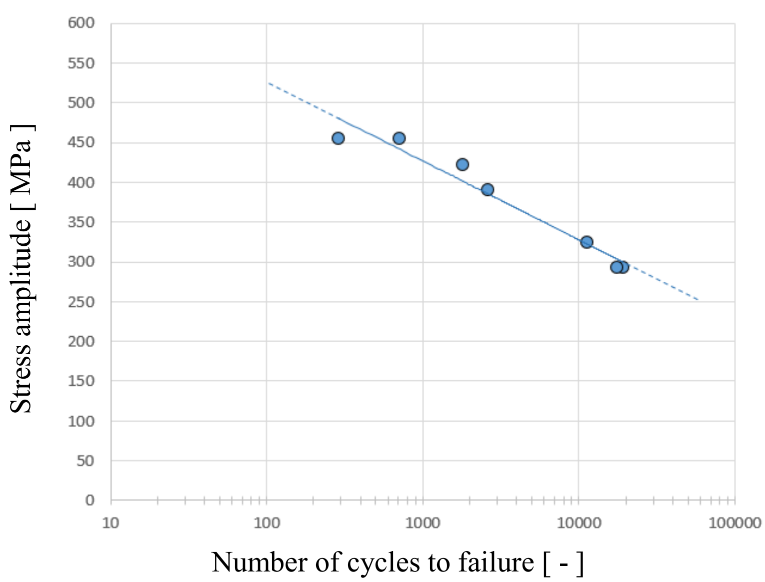


Figure 8. S-N curve of fatigue tests.

**ESTIMATION OF INTEGRATED ENERGY**

General solution of a mass point model is as follows,

$$x = \beta \sin(\delta - \omega t) \tag{3}$$

where, x is displacement, β is displacement amplitude, δ is phase, ω is angular frequency and t is time. In case that angular frequency is assumed to be resonance frequency, displacement amplitude β and phase δ are developed as follows,

$$\beta = \frac{\alpha}{2\zeta\omega^2}, \quad \delta = \frac{\pi}{2} \tag{4}$$

where, α is input acceleration, ζ is damping ratio. As results, displacement x and velocity $\dot{x}$ are as follows,

$$x = \beta \sin\left(\frac{\pi}{2} - \omega t\right) \tag{5}$$

$$\dot{x} = -\omega\beta \cos\left(\frac{\pi}{2} - \omega \mathrm{t}\right) = -\omega\beta \sin \omega t \tag{6}$$

Integrated energy at time $t_e$ is developed by input acceleration α, displacement x and velocity $\dot{x}$ and it is estimated as follows,

$$\begin{aligned} m\int_0^{t_e} \alpha \sin \omega t \cdot \dot{x}\, dt &= \frac{\alpha^2 m}{2\zeta\omega}\int_0^{t_e} \sin^2 \omega t\, dt \\ &= \frac{\alpha^2 m}{2\zeta\omega}\left[\frac{1}{2}t - \frac{\omega}{4}\sin 2\omega t\right]_0^{t_e} \\ &= \frac{\alpha^2 m}{2\zeta\omega}\left\{\left(\frac{1}{2}t_e - \frac{\omega}{4}\sin 2\omega t_e\right) - \left(\frac{1}{2}0 - \frac{\omega}{4}\sin 2\omega \cdot 0\right)\right\} \\ &\approx \frac{\alpha^2 m}{4\zeta\omega}t_e \end{aligned} \tag{7}$$

From above equation, integrated energy at time $t_e$ is estimated by mass, resonance frequency and damping ratio of system. Input acceleration is known value.

On the other hand, the energy at failure time are estimated by results of fatigue test. It means that the energy at failure time is tried to estimate without vibration test. As results of fatigue test, relationship between stress amplitude and number of cycle to failure has already estimated as equation (2). Failure time te is defined by number of cycles to failure and frequency of specimen, 5.3 Hz, and it is as follows.

$$t_e = \frac{N_a}{f} \tag{8}$$

Specimen of vibration test is defined as cantilever and maximum stress is shown by modulus of section Z and modulus of plastic section $Z_p$. The equations are as follows,

$$\sigma_m = \frac{M}{Z} = \frac{Fl}{Z} = \frac{m\ddot{x}l}{Z} = ml\frac{6}{ab^2}\ddot{x} \tag{9}$$

$$\sigma_m = \frac{M}{Z_P} = \frac{Fl}{Z_P} = \frac{m\ddot{x}l}{Z_P} = ml\frac{4}{ab^2}\ddot{x} \tag{10}$$

Integrated energy with modulus of section Z is estimated by equation (2), (7), (8) and (9), and the energy with modulus of plastic section $Z_p$ is estimated by equation (2), (7), (8) and (10). Both estimated curves and results of vibration test are shown in Figure 9. Experimental results are plotted among the curves.

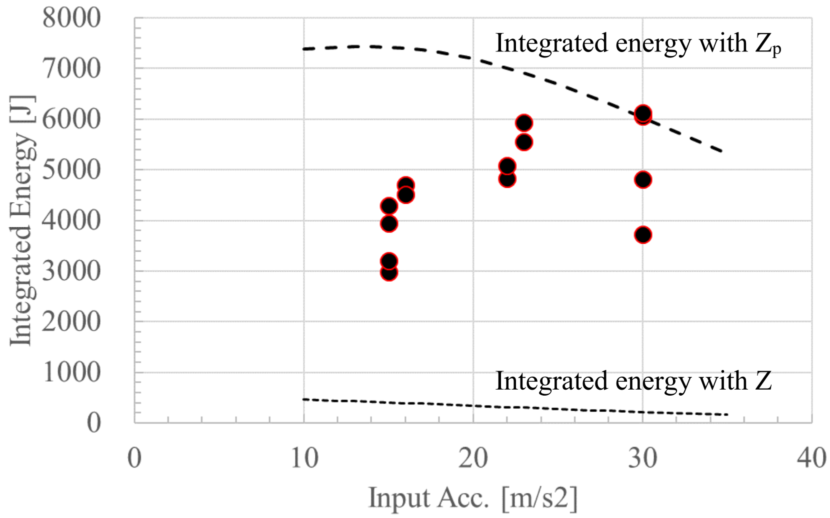


Figure 9. Relationship between integrated energy at failure and input acceleration.

## CONCLUSION

In this study, fatigue tests and vibration tests were carried out to discuss the failure probability evaluation method with integrated energy. Fatigue tests were carried out to obtain relation of stress amplitude and number of cycles to failure, and equation of the relation was approximated up to stress amplitude of 450 MPa. Vibration tests were carried out to estimate integrated vibration energy at failure, and relationship between the energy at failure and input acceleration were shown. Integrated energy is estimated by results of fatigue tests, and the energy is compared with the integrated vibration energy by results of vibration tests. Integrated energy at failure time by vibration tests were among estimated energy curves by results of fatigue tests. Furthermore, accurate estimate of the energy at failure are discussed by these results of this study.

## REFERENCES

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